

Instrumentation

Fundamentals of Radio Interferometry



Griffin Foster

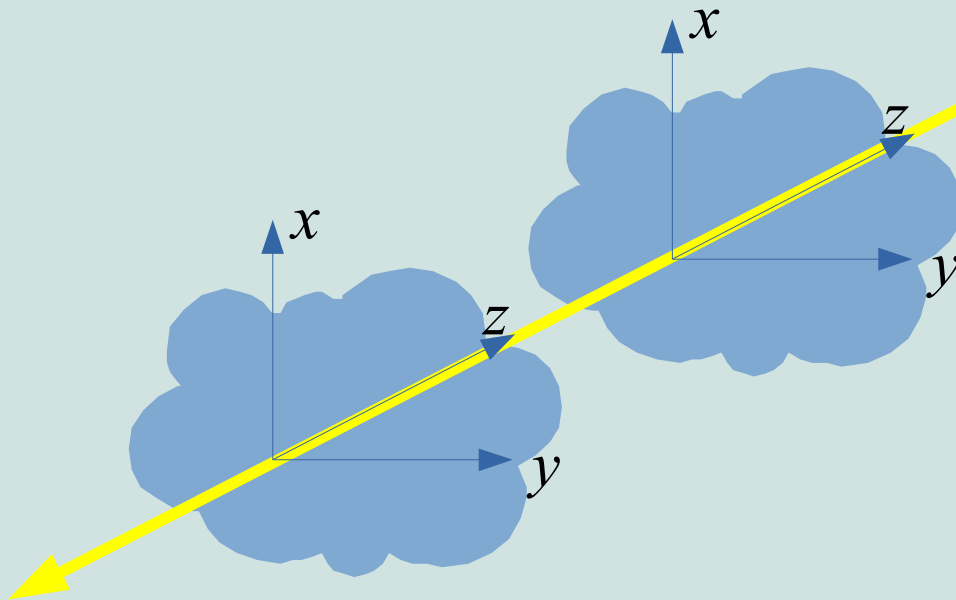
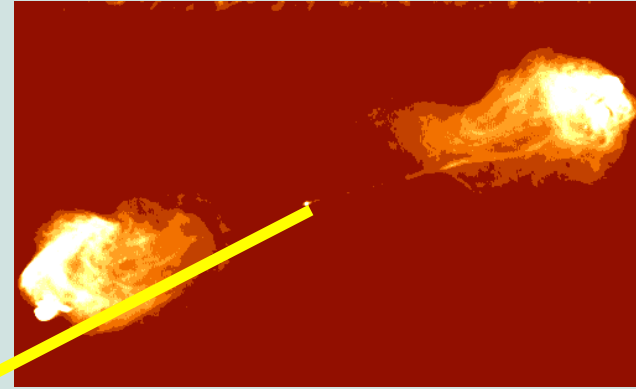
SKA SA/Rhodes University

NASSP 2016

Jones Chains

Multiple propagation effects
can be described by chaining up
Jones matrices:

$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$



$$\mathbf{e}' = \begin{bmatrix} e'_x \\ e'_y \end{bmatrix}$$

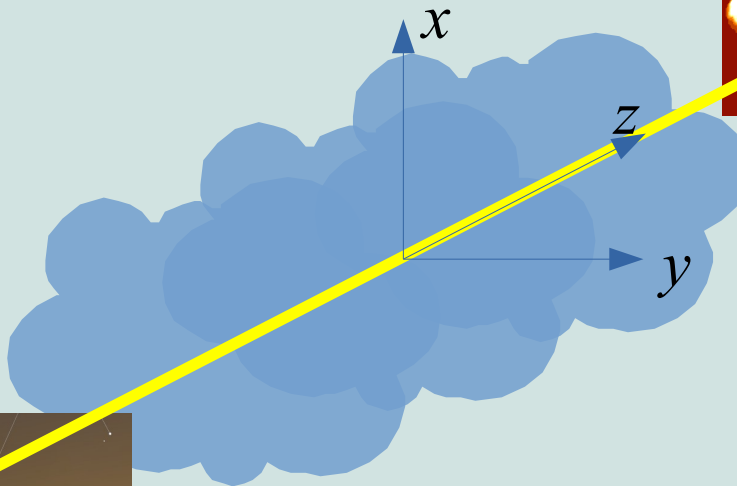
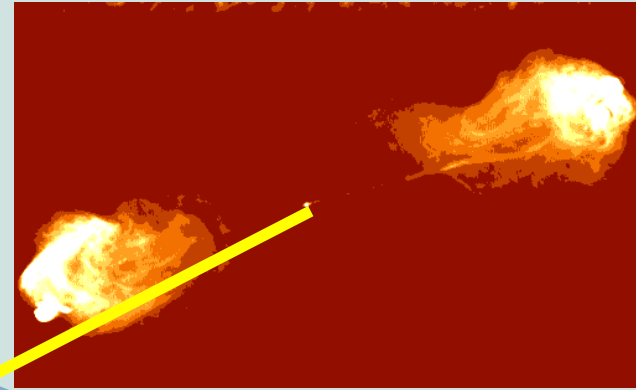
$$\mathbf{e}' = \mathbf{J}_2 \mathbf{J}_1 \mathbf{e}$$

Enter The Antenna

A dual-receptor feed measures two complex voltages (polarizations):

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$



We may further assume the voltage conversion process is also linear. Therefore we have:

$$\mathbf{v} = \mathbf{J}_n \mathbf{J}_{n-1} \dots \mathbf{J}_1 \mathbf{e} = \mathbf{J}_{\text{sys}} \mathbf{e}$$

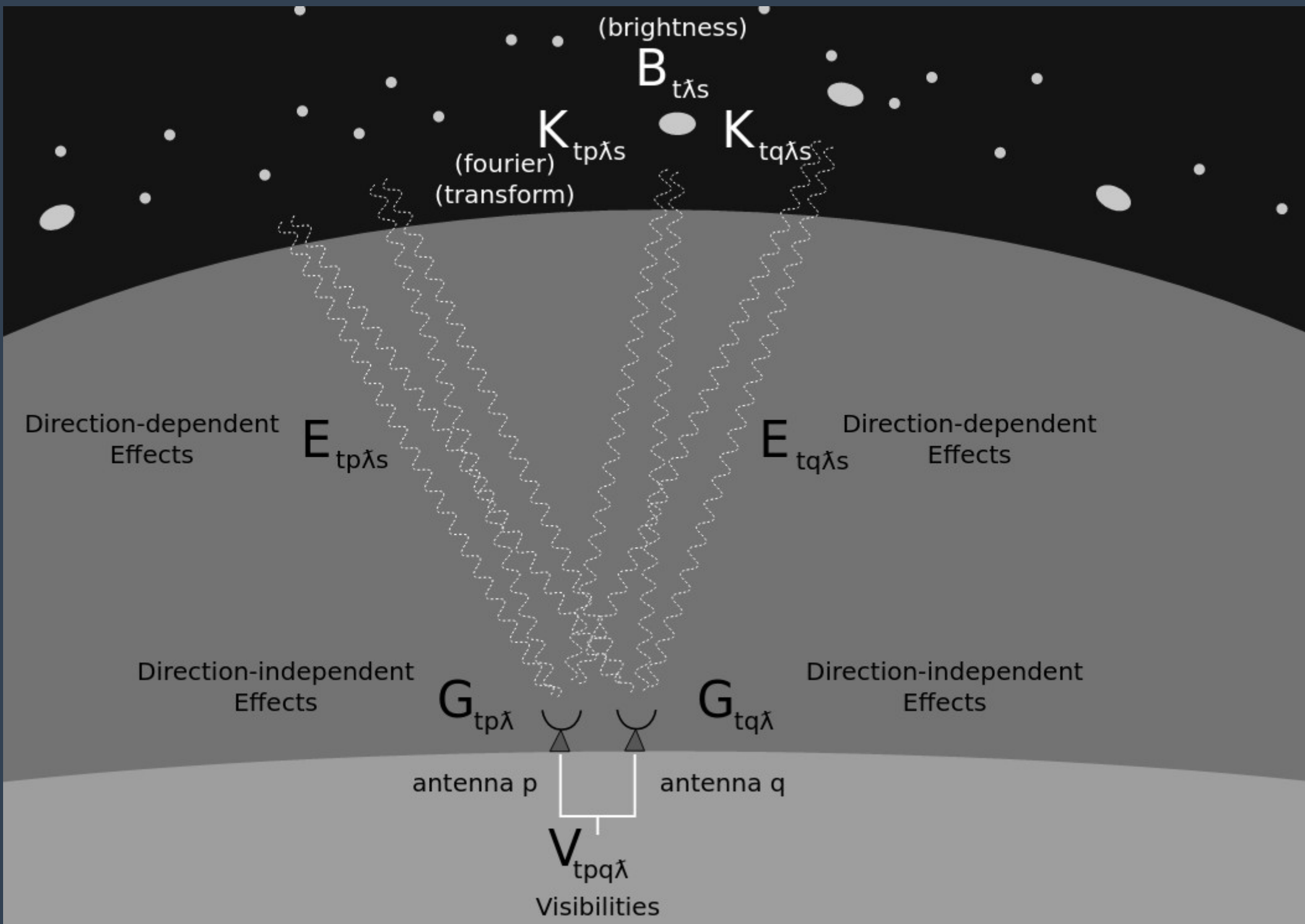


Jones Sequences

$$\mathbf{v} = \mathbf{J}_n \mathbf{J}_{n-1} \dots \mathbf{J}_1 \mathbf{e} = \mathbf{J}_{\text{sys}} \mathbf{e}$$

$$\mathbf{J}_{\text{sys}} = \mathbf{G} \mathbf{B} \mathbf{D} \mathbf{E} \mathbf{K} \mathbf{P} \mathbf{Z} \mathbf{F}$$

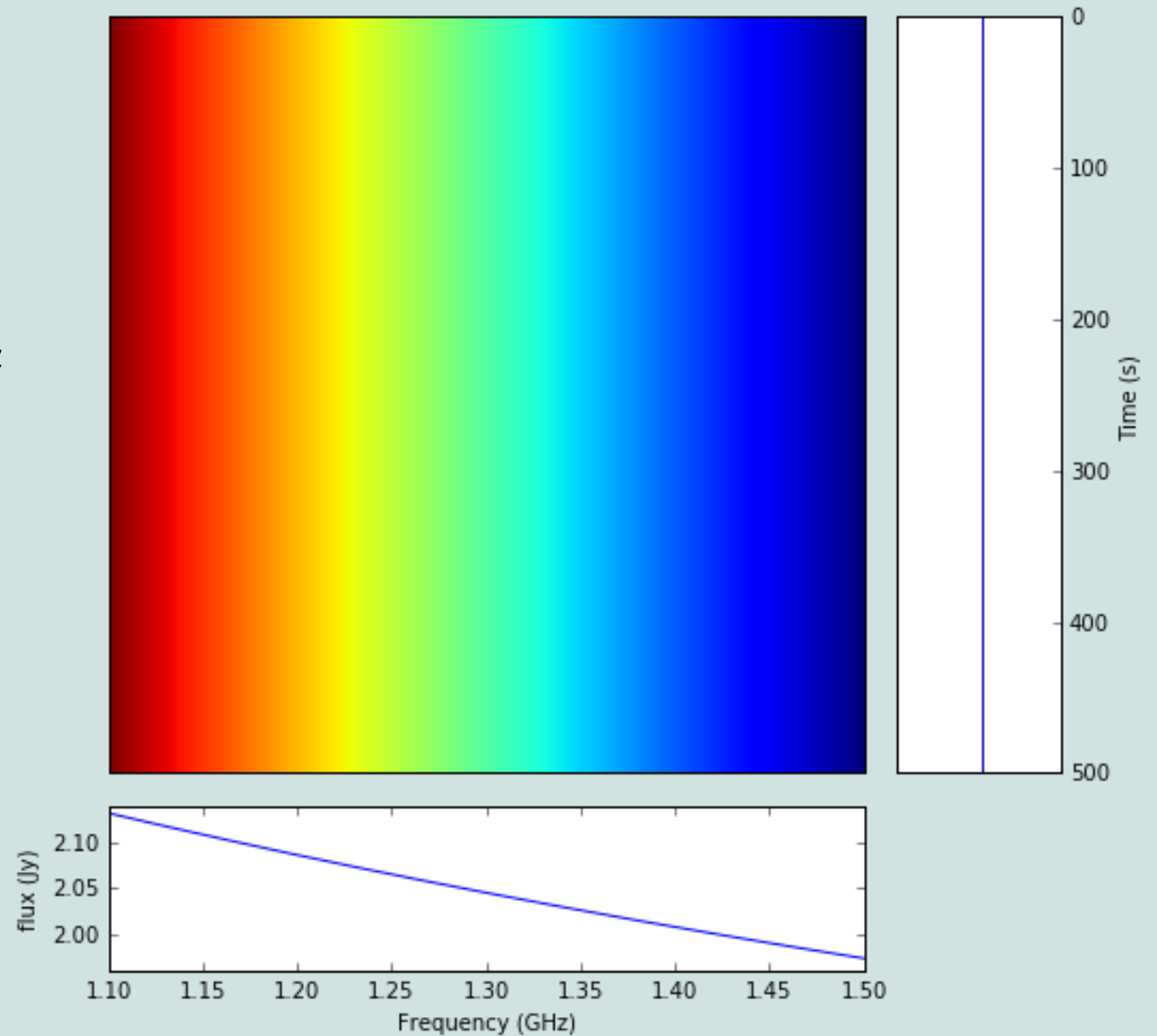
- This is just an example!
- Order is important: matrices don't (in general) commute
 - Must follow physical order of propagation effects
- Some specific matrices do commute
 - Scalar matrix (K-Jones) commutes with everything
 - Diagonal matrices commute among themselves
 - Rotation matrices commute among themselves



An Idealized Source

Idealized Source Spectrum

$$I(\nu) = I_0 \left(\frac{\nu}{\nu_0} \right)^{-\alpha}$$



Noise as Temperature

Johnson-Nyquist noise source (thermal source):

$$P = k_B T \Delta\nu$$

For a fixed bandwidth, rearranging:

$$T = \frac{P}{k_B \Delta\nu}$$

System Temperature

$$T_{\text{sys}} = T_{\text{sky}} + T_{\text{atmosphere}} + T_{\text{spillover}} + T_{\text{rx}} + \dots$$

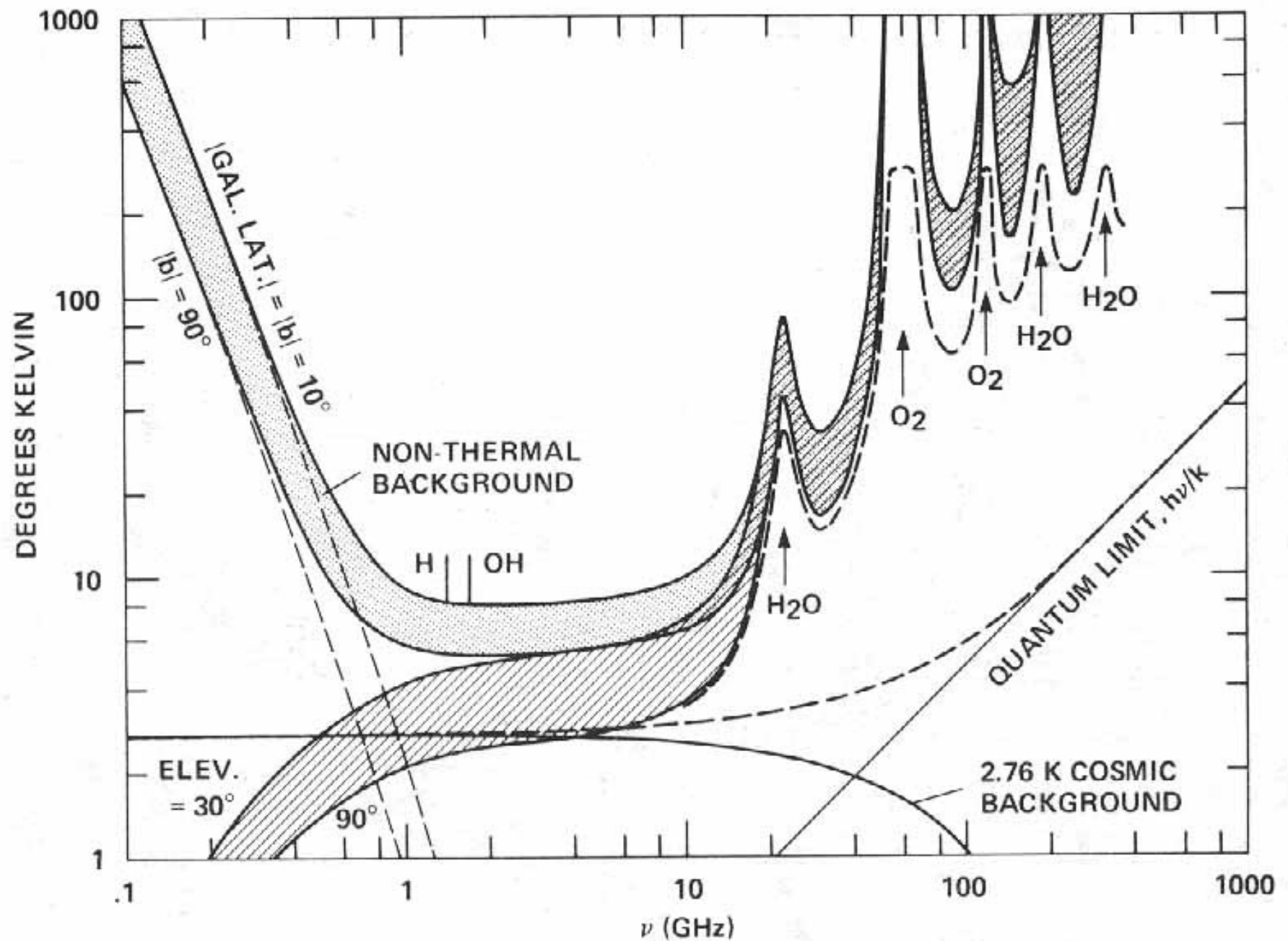
T_{sky} : radio sky background (synchrotron, CMB (2.76K), ...)

$T_{\text{atmosphere}}$: atmospheric foregrounds (important at mm wavelengths)

$T_{\text{spillover}}$: pick-up of ~300K ground in the side and back-lobes

T_{rx} : receiver temperature from the Friis Cascade Noise Equation

System Temperature



System Temperature

T_{passive} : passive components (cables, connectors, OMT) before the LNA

T_{LNA} : Low-Noise Amplifier temperature

T_{amp} : secondary amplification/attenuation temperature

G_{LNA} : gain of the LNA

G_{feed} , G_{passive} : feed and passive 'gain' (related to efficiency)

$$T_{rx} = T_{feed} + \frac{T_{passive}}{G_{feed}} + \frac{T_{LNA}}{G_{feed}G_{passive}} + \frac{T_{amp}}{G_{feed}G_{passive}G_{LNA}} + \dots$$

Radiometer Equation

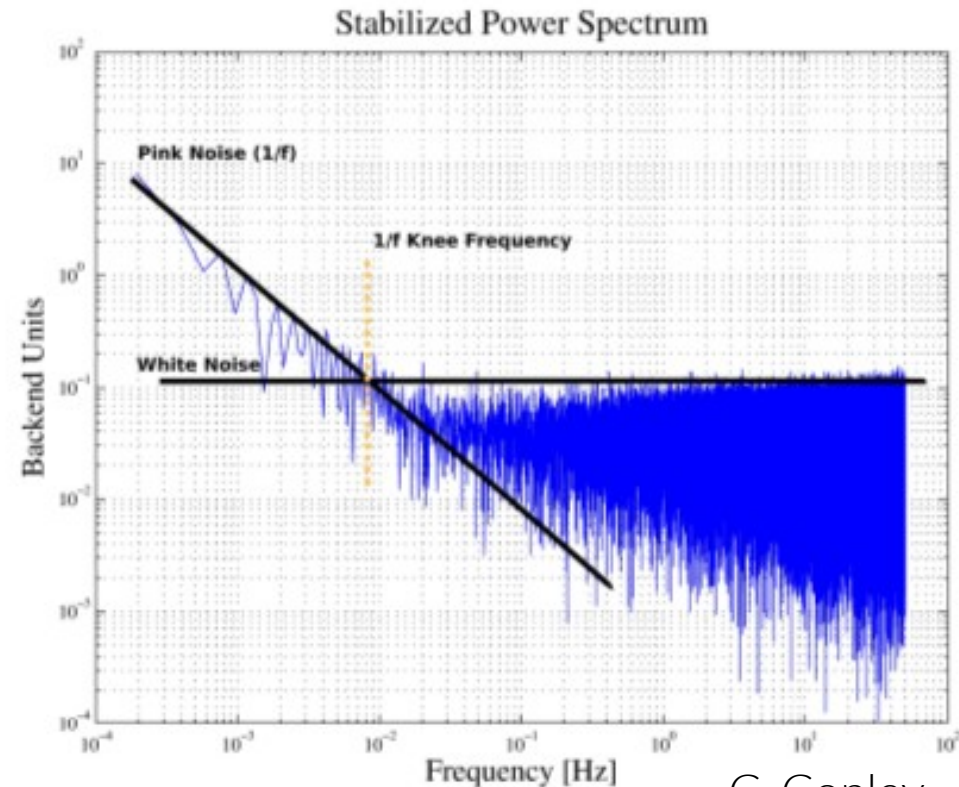
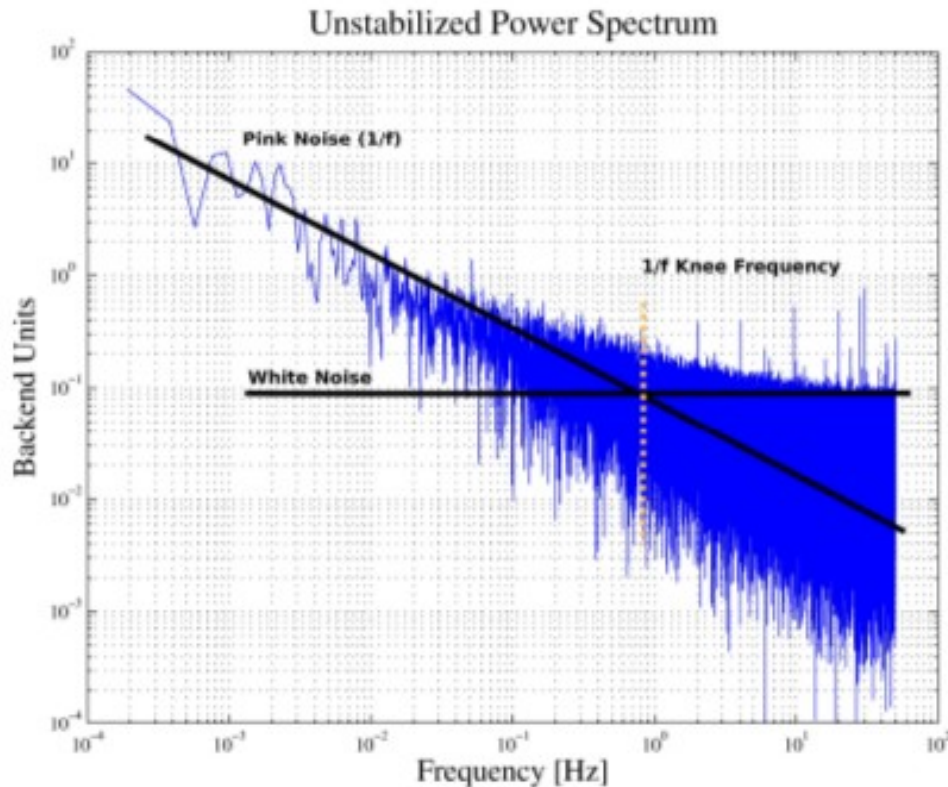
$$\sigma_T = \frac{T_{\text{sys}}}{\sqrt{\Delta\nu\tau}}$$

σ_T : residual (root-mean-square) in the measurement

$\Delta\nu$: bandwidth of observation (Hz)

τ : integration time (seconds)

→ the smaller the T_{sys} the shorter the required observation time

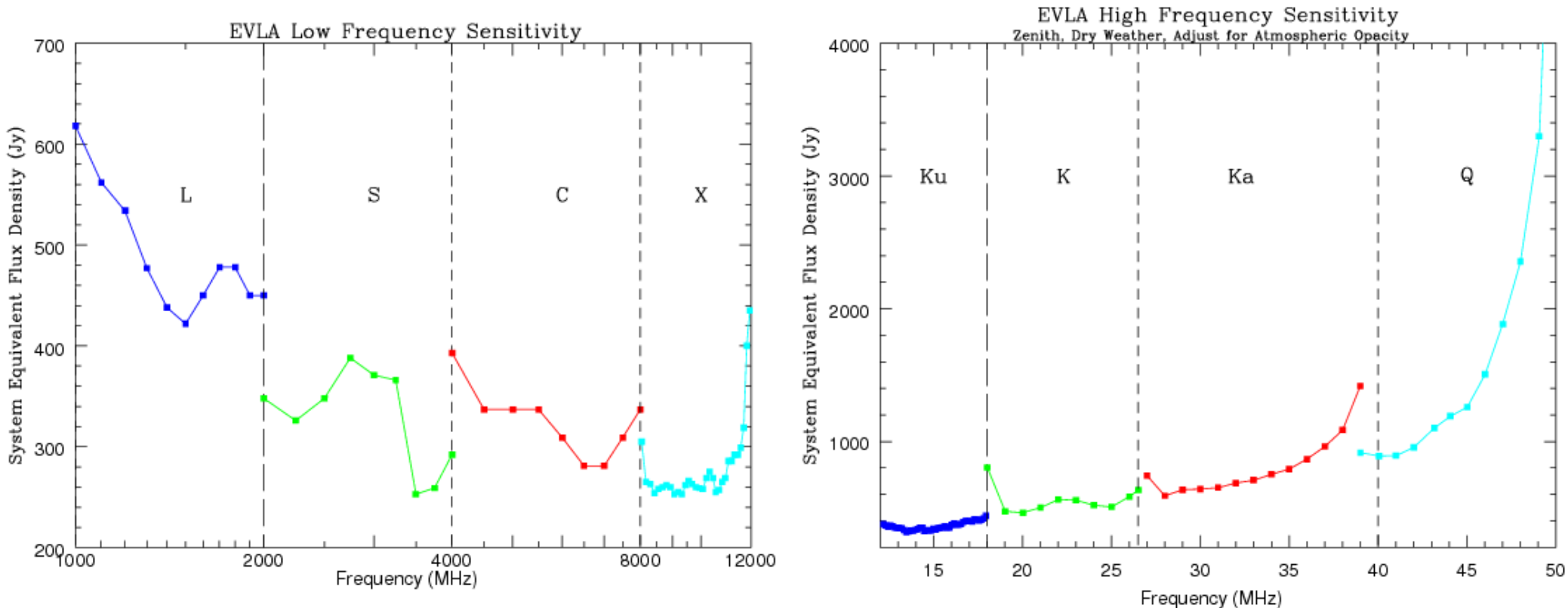


System Equivalent Flux Density (SEFD)

flux density of a radio source that doubles the system temperature

$$\text{SEFD} = \frac{T_{\text{sys}}}{G_{\text{eff}}} = \frac{2k_B\eta T_{\text{sys}}}{A_{\text{eff}}}$$

Very Large Array SEFD



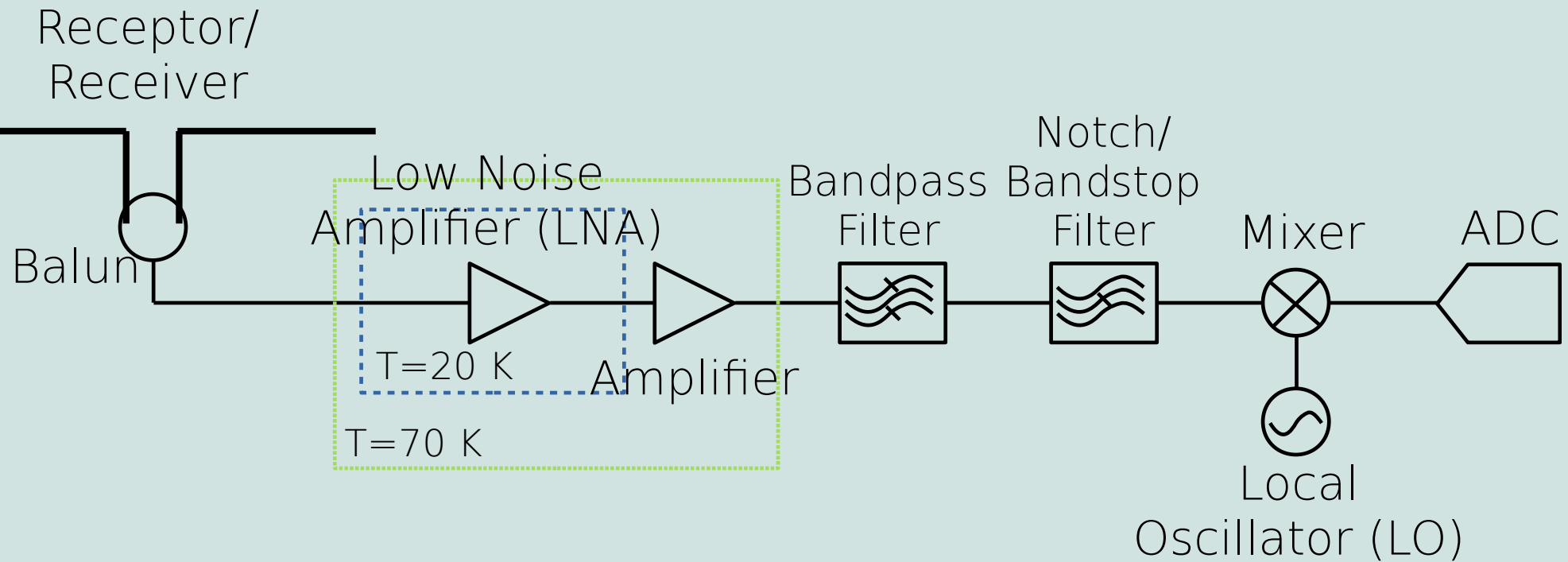
Analogue Front-end (G- and B-Jones)

Amplitude and attenuation due to the system electronics.

$$\mathbf{G}'(t, \nu) \approx \mathbf{G}(t) \cdot \mathbf{B}(\nu) = \begin{pmatrix} G_x(t) & 0 \\ 0 & G_y(t) \end{pmatrix} \cdot \begin{pmatrix} B_x(\nu) & 0 \\ 0 & B_y(\nu) \end{pmatrix}$$

The total, *time*- and *frequency*-dependent gain Jones matrix is often split into a gain (**G**) and bandpass (**B**) Jones matrices.

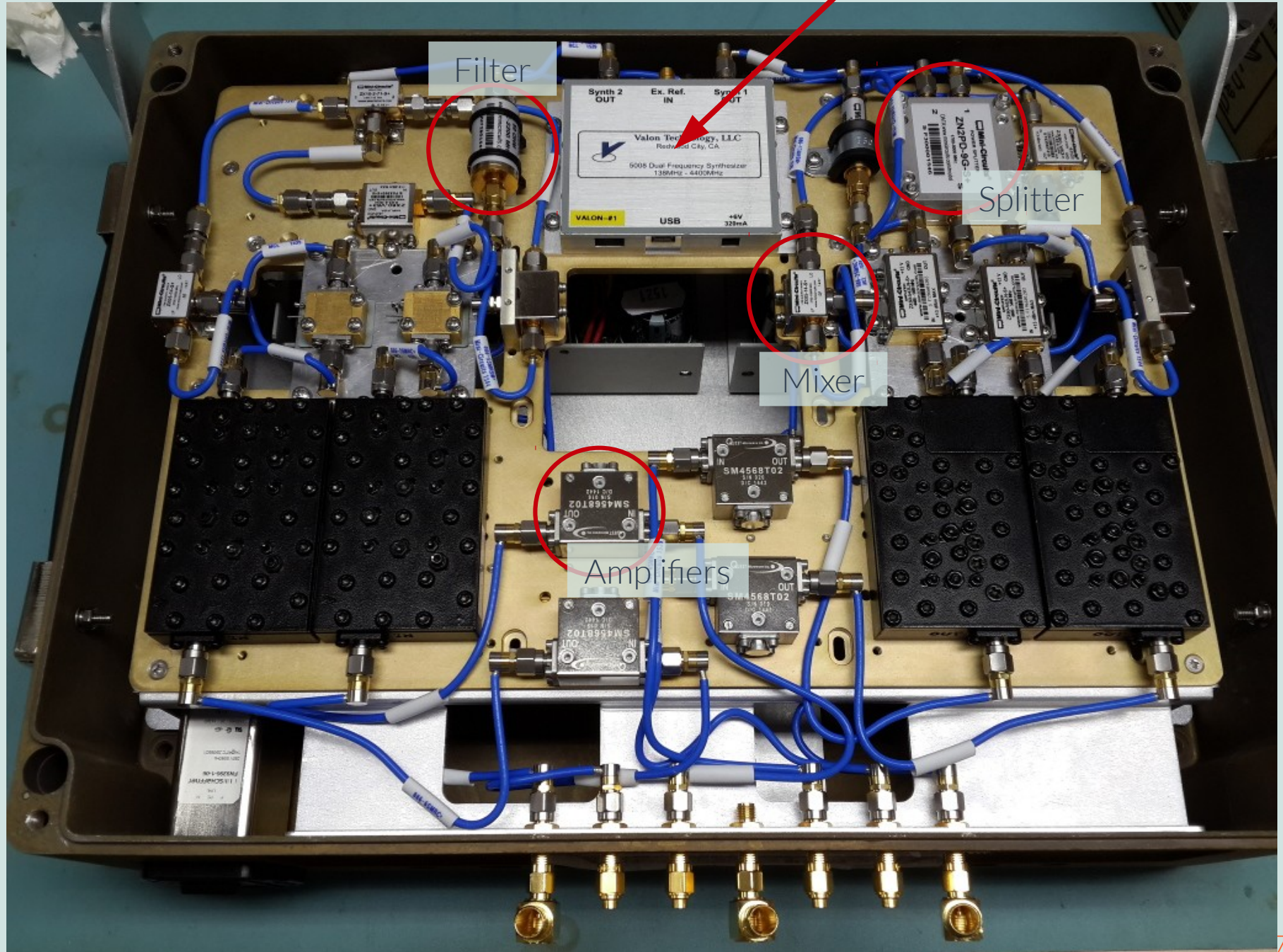
Analogue Front-end (G- and B-Jones)



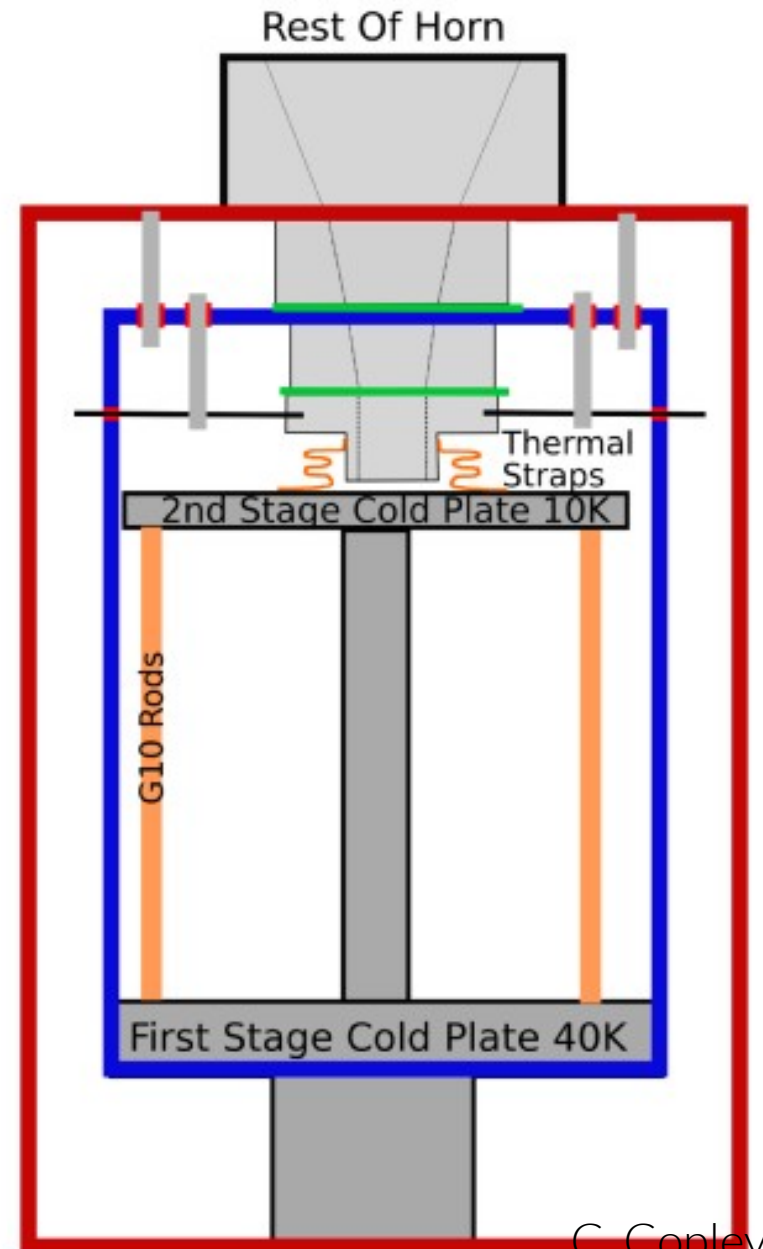
$$T_{\text{rx}} = T_{\text{feed}} + \frac{T_{\text{passive}}}{G_{\text{feed}}} + \frac{T_{\text{LNA}}}{G_{\text{feed}}G_{\text{passive}}} + \frac{T_{\text{amp}}}{G_{\text{feed}}G_{\text{passive}}G_{\text{LNA}}} + \dots$$

Analogue Front-end (G- and B-Jones)

Local Oscillator (LO)



Cryostat



C. Copley

Decibel Units

Power:

$$P_{dB} = 10 \log_{10} \left(\frac{P}{P_0} \right)$$

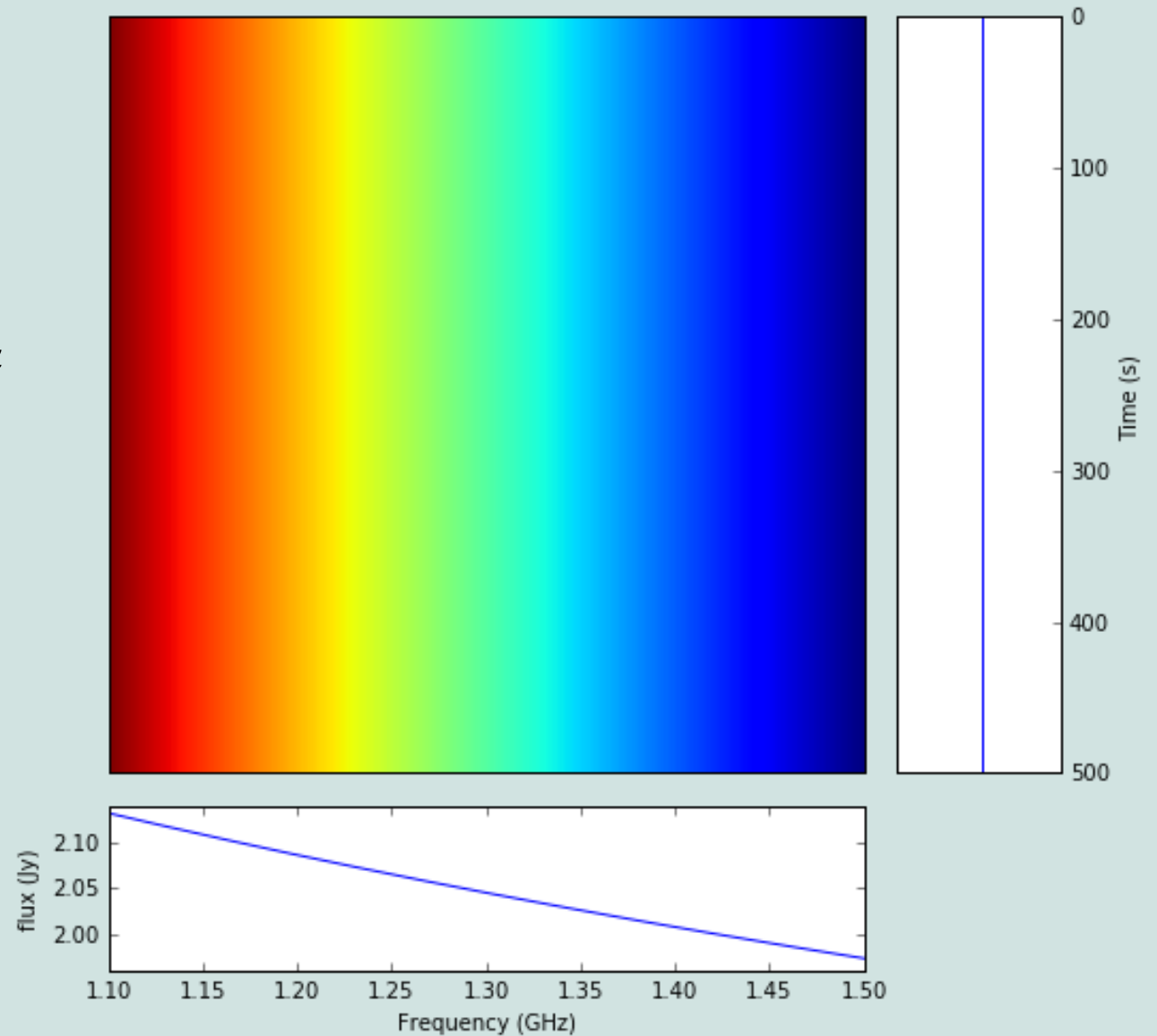
Voltage Amplitude:

$$P_{dB} = 20 \log_{10} \left(\frac{V}{V_0} \right)$$

An Idealized Source

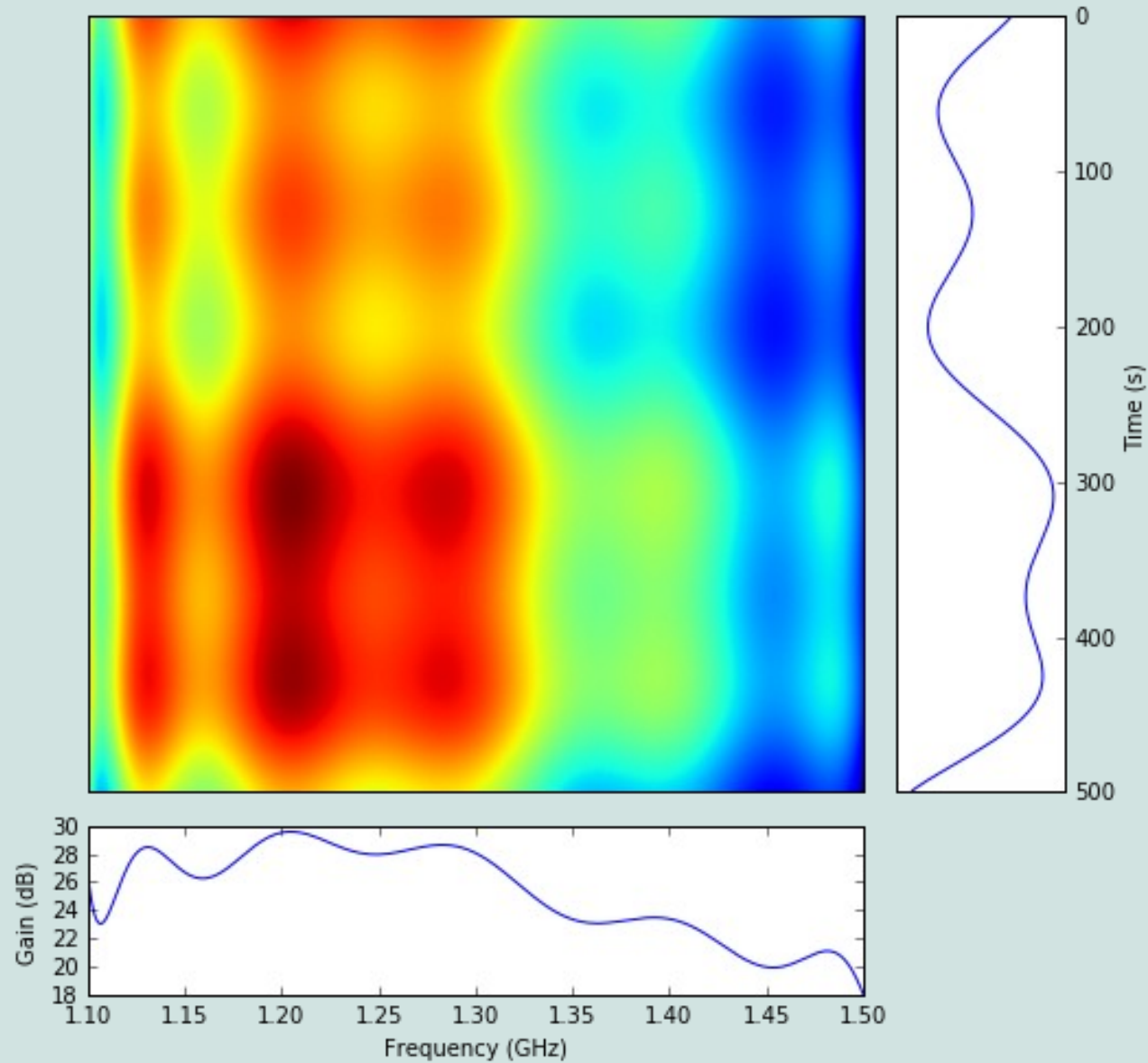
Idealized Source Spectrum

$$I(\nu) = I_0 \left(\frac{\nu}{\nu_0} \right)^{-\alpha}$$

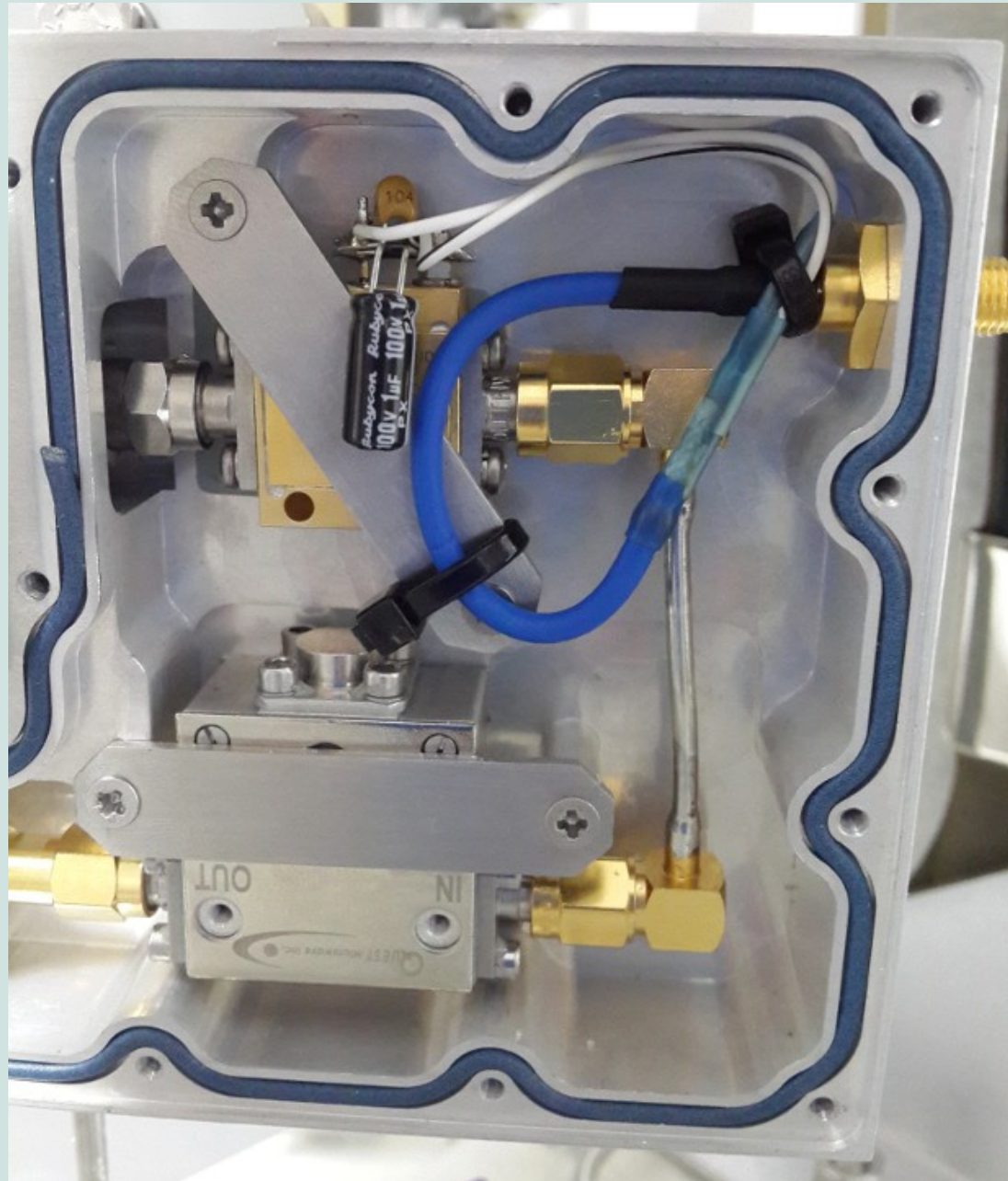


Low Noise Amplifier (LNA) Response

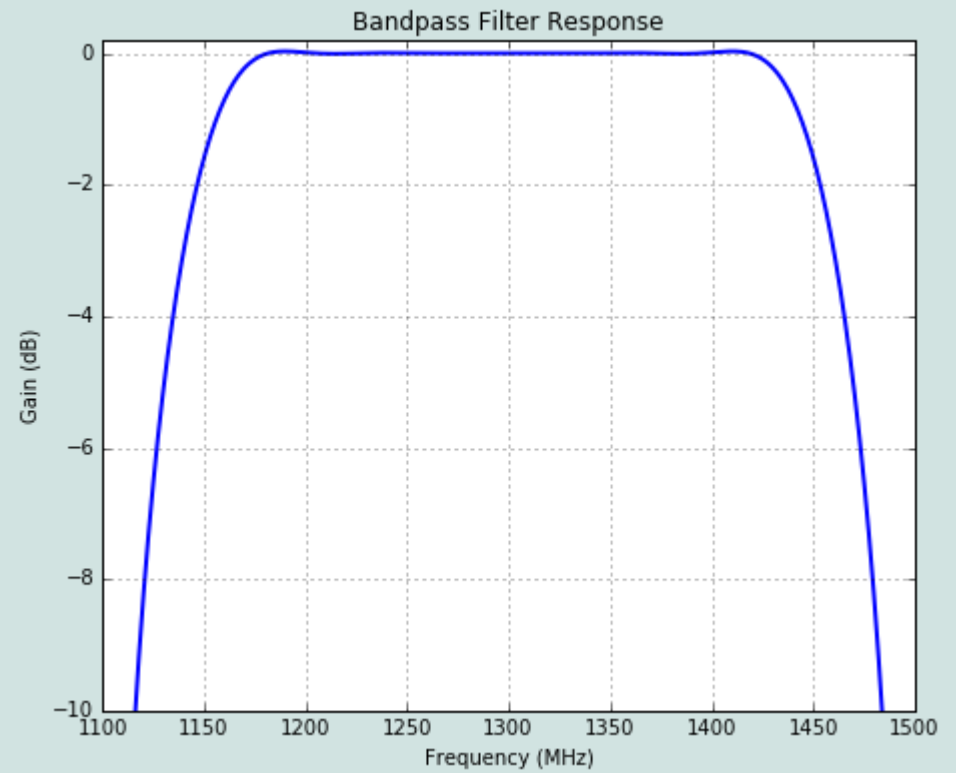
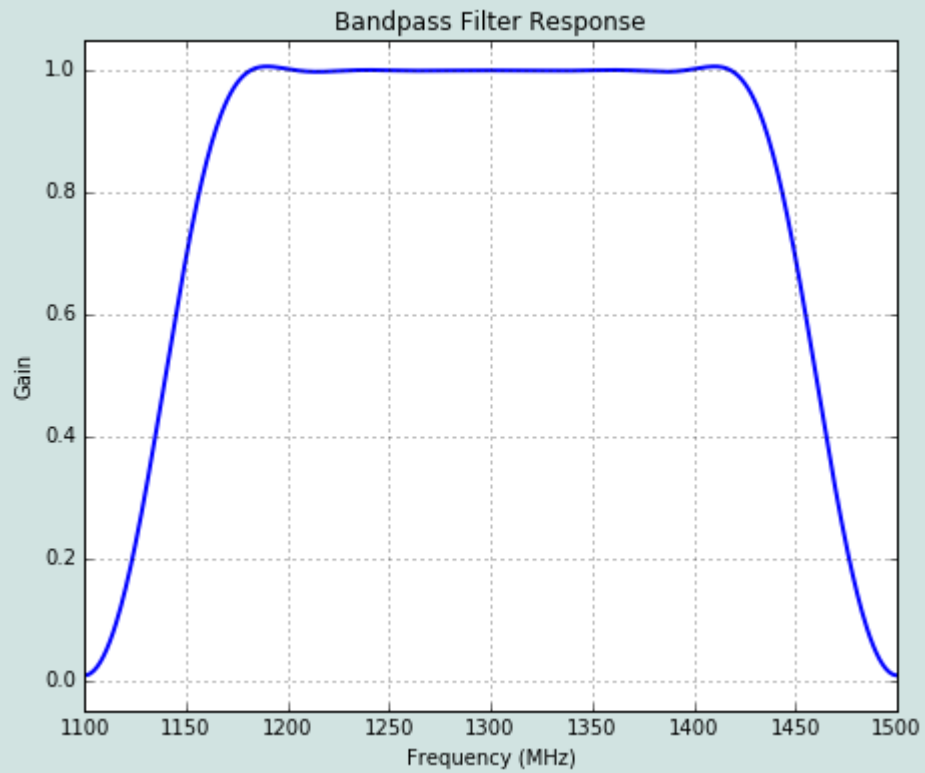
LNA Response



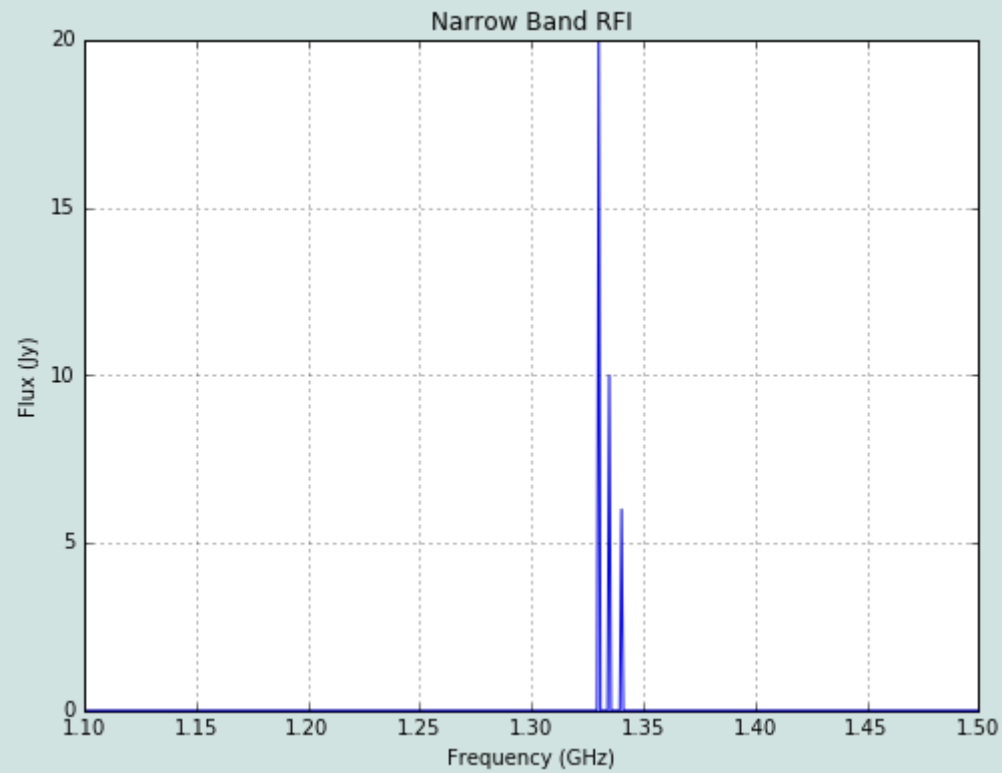
Low Noise Amplifier (LNA)



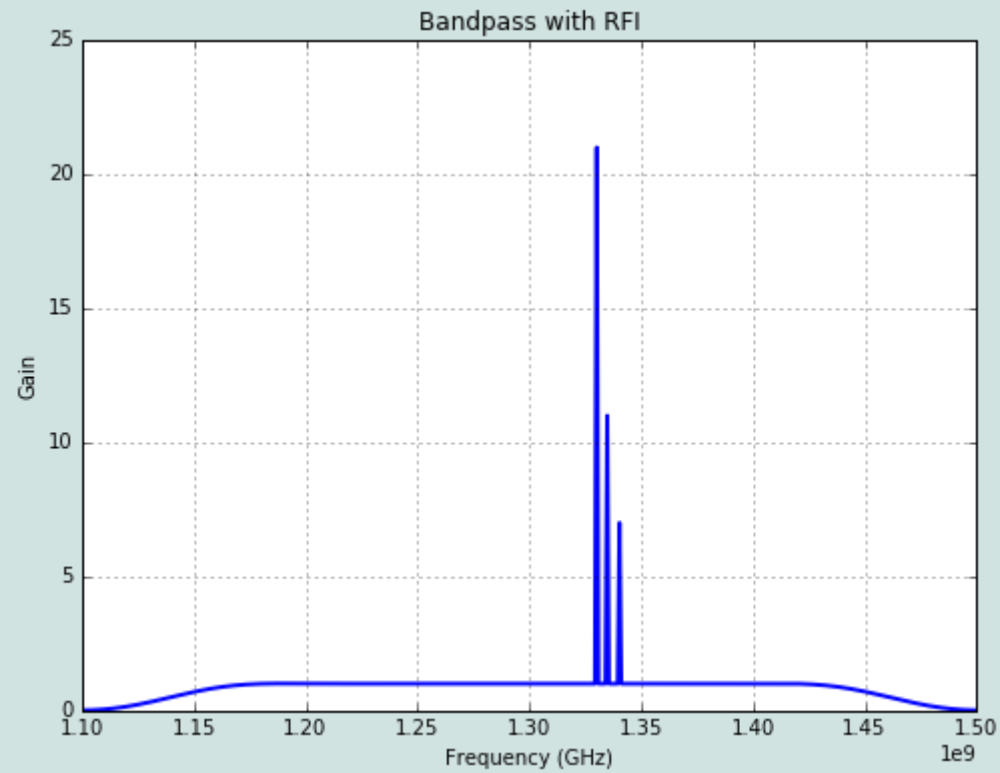
Bandpass Filter



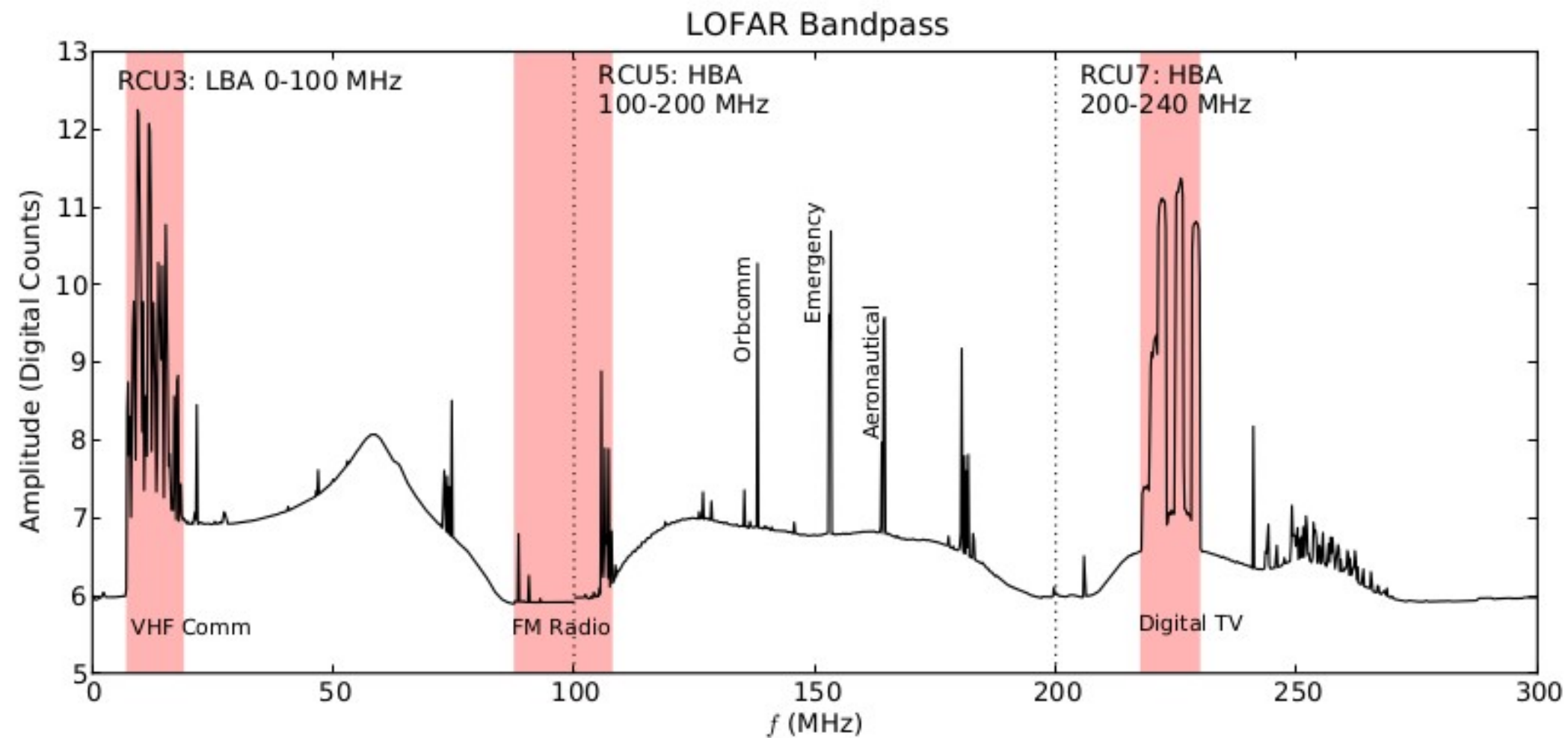
Narrow-band Radio Frequency Interference (RFI)



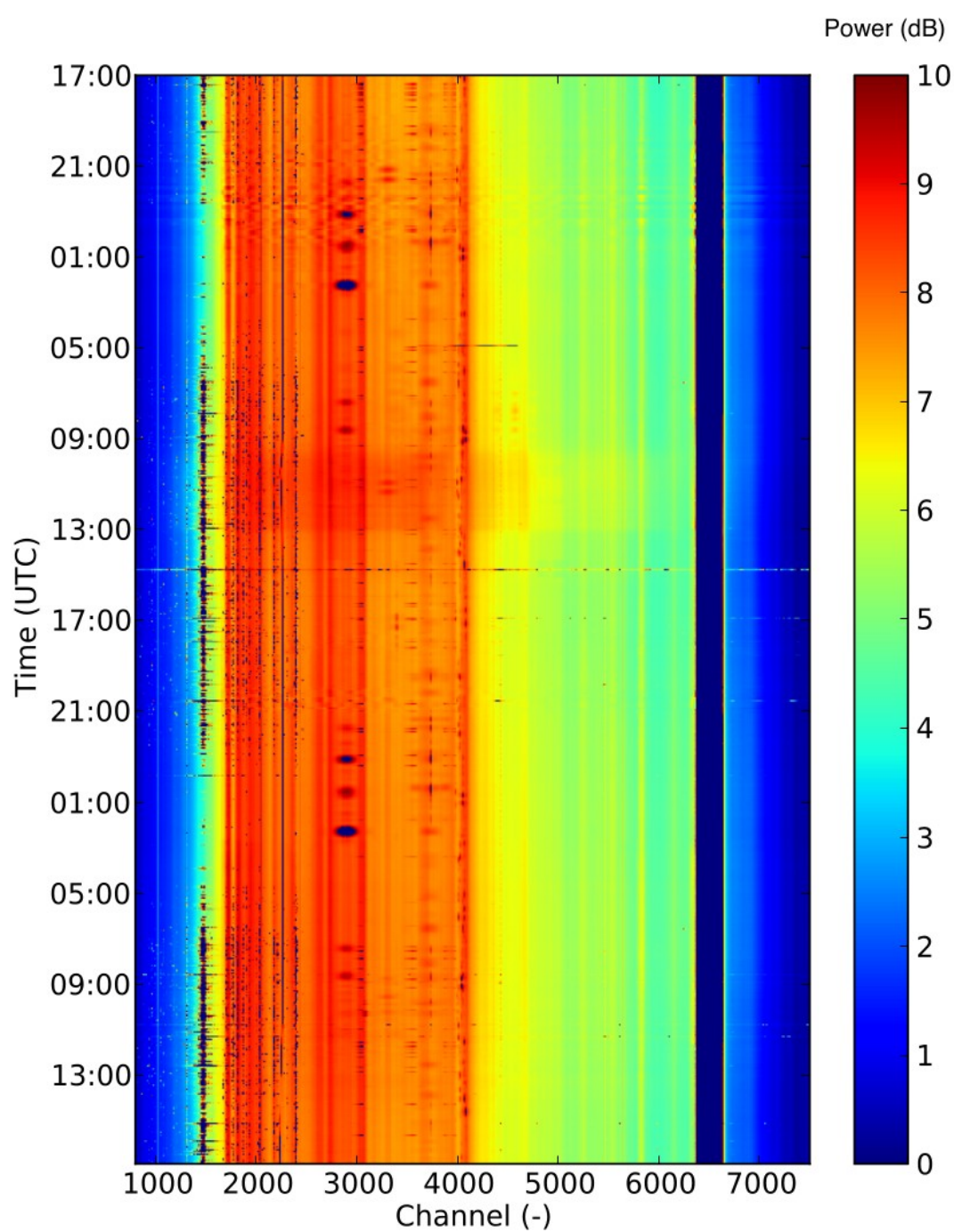
Narrow-band Radio Frequency Interference (RFI)



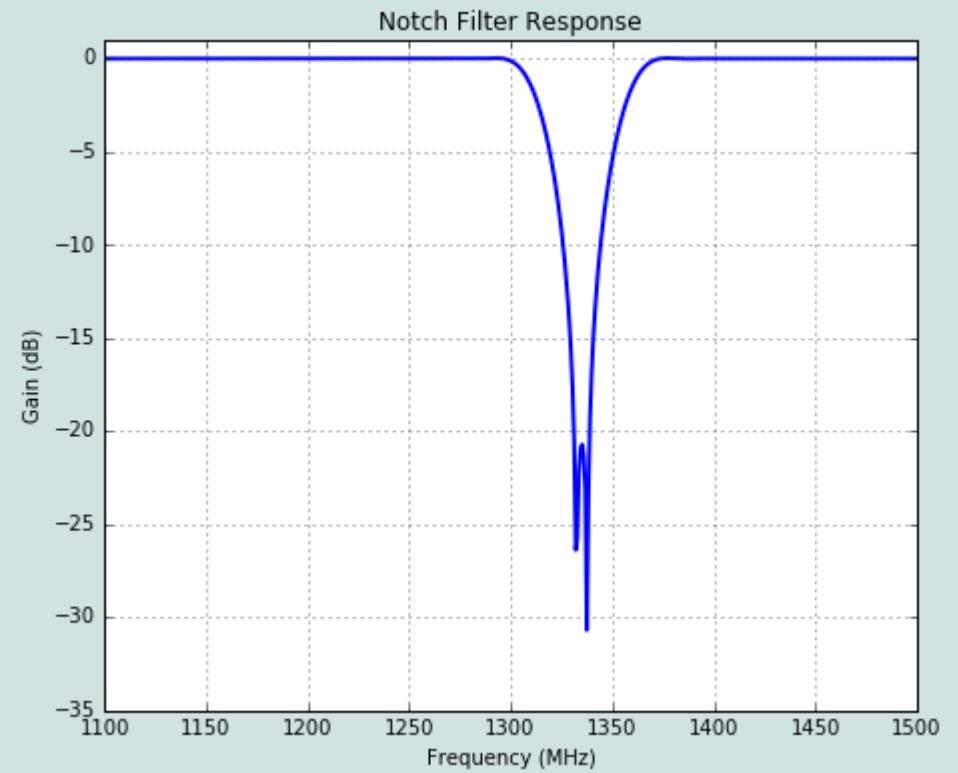
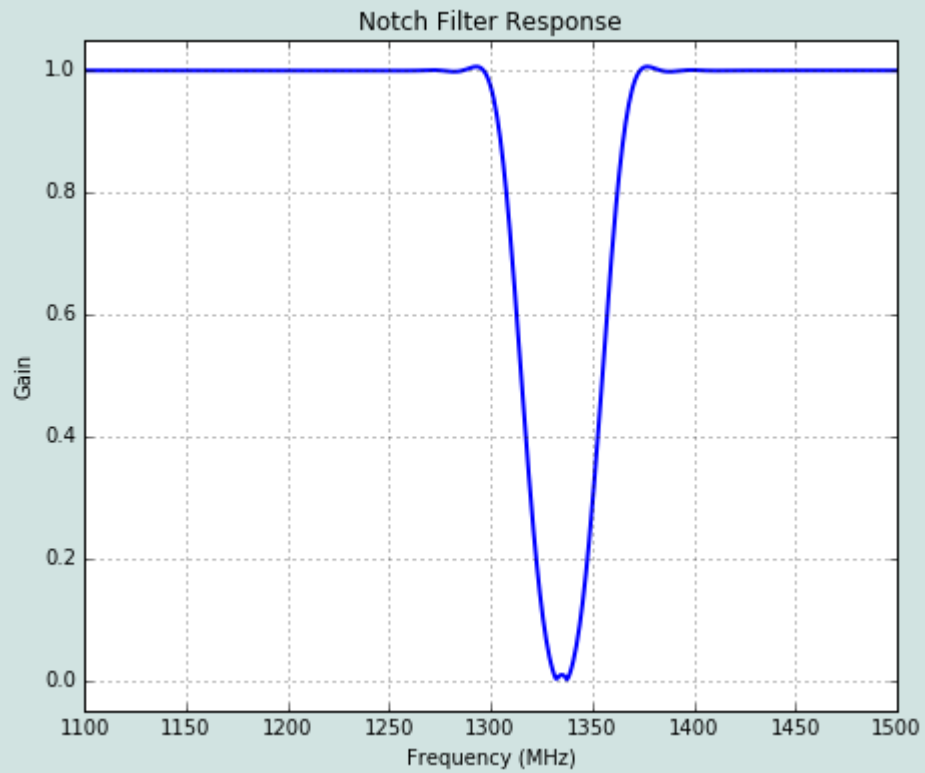
Narrow-band Radio Frequency Interference (RFI)



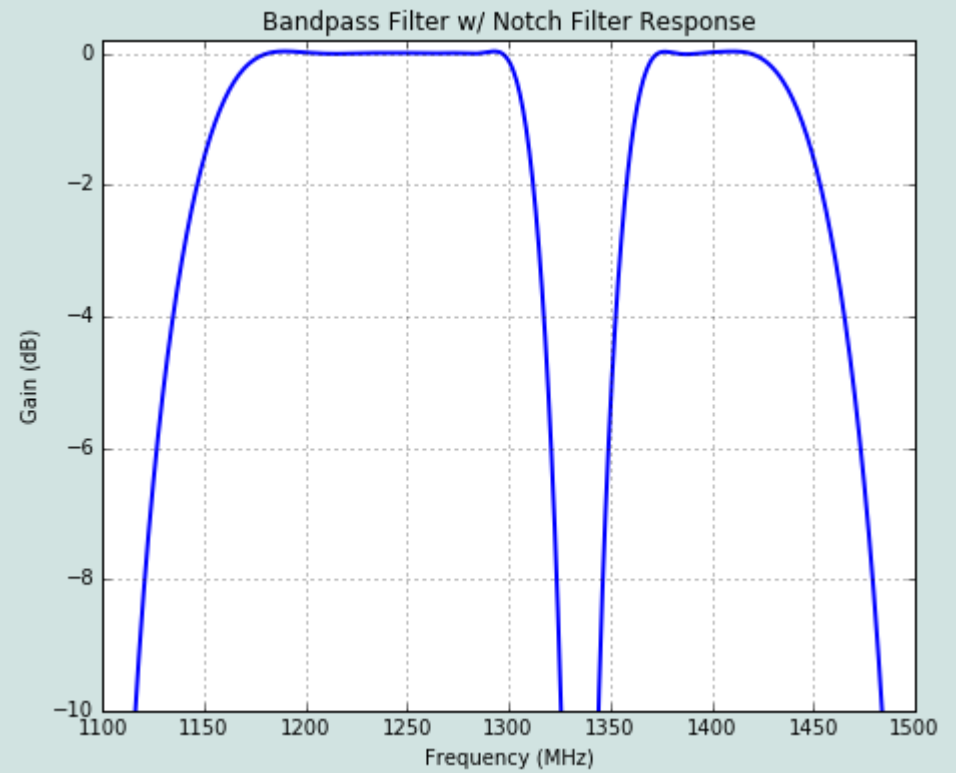
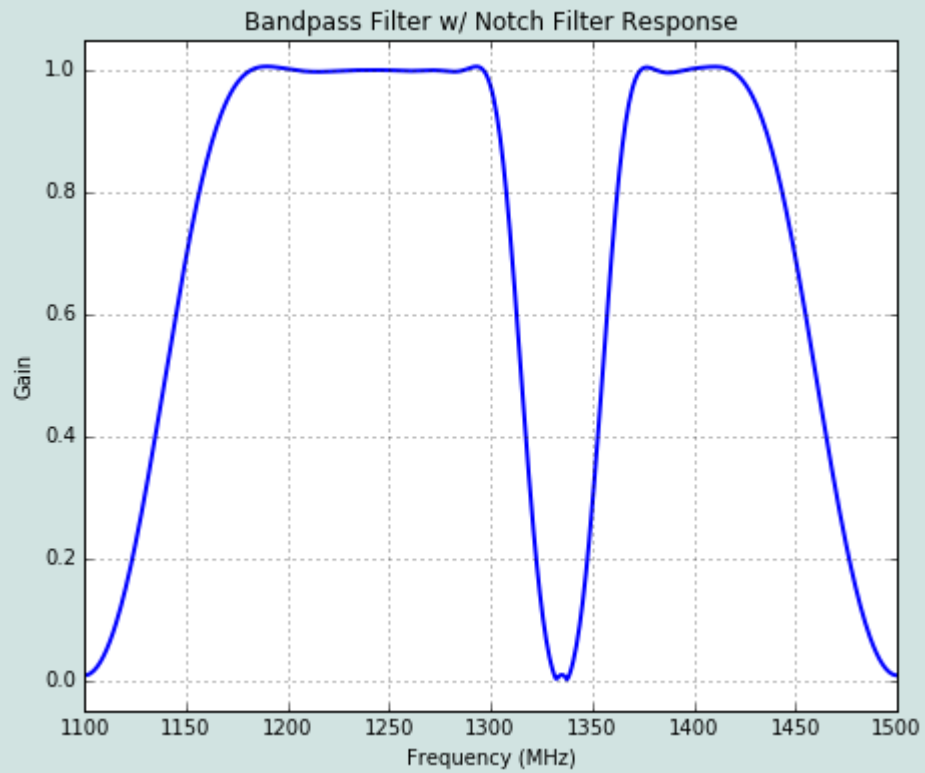
Narrow-band Radio Frequency Interference (RFI)



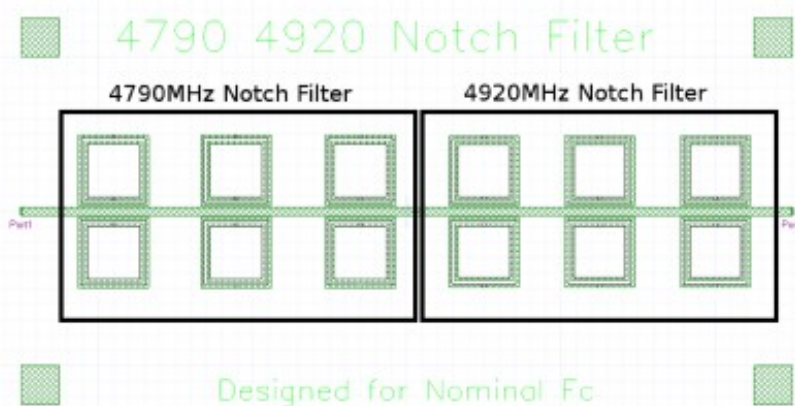
Notch/Bandstop Filter



Notch/Bandstop Filter



Notch/Bandstop Filter



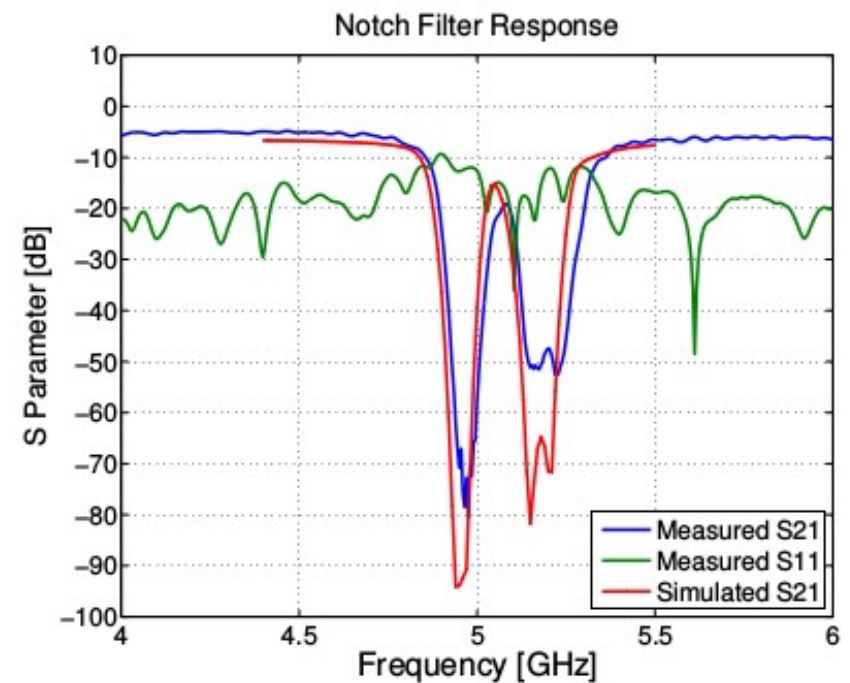
(a) The 4.79 GHz and 4.92 GHz notch filter



(b) The 5.18 GHz and 5.24 GHz notch filter



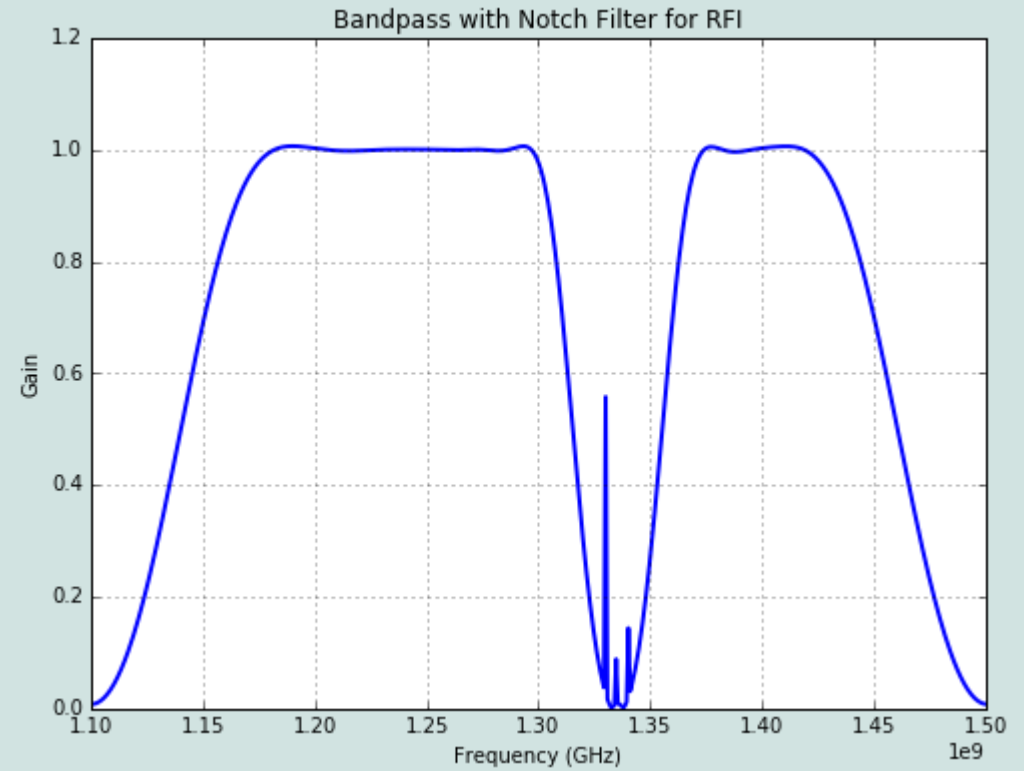
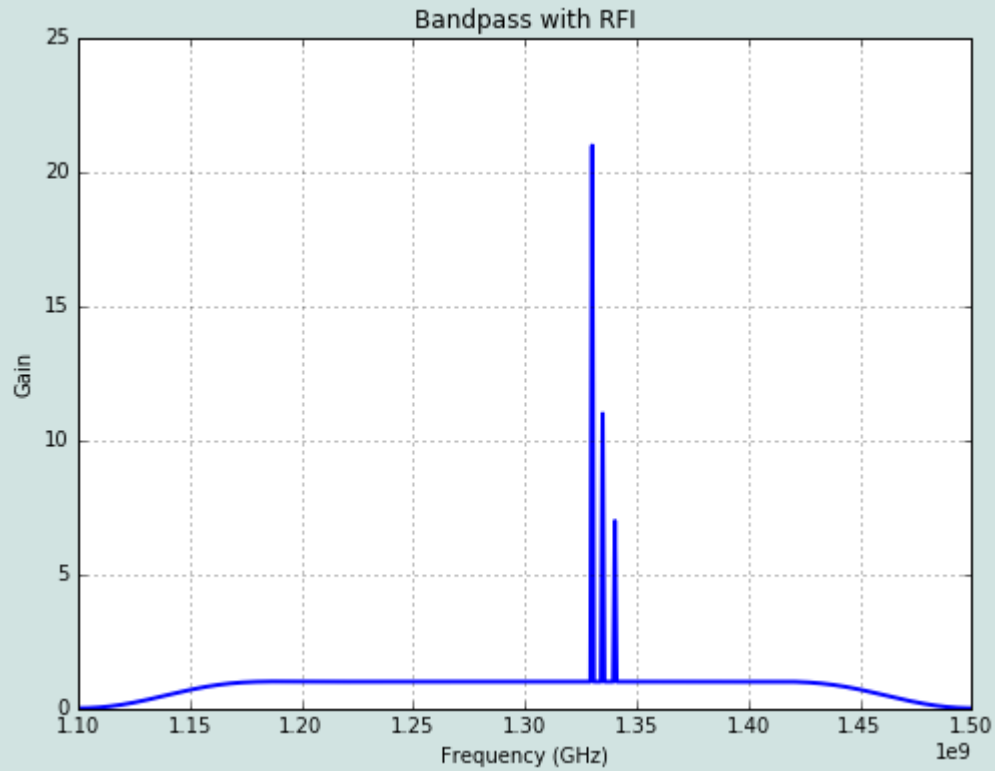
(c) Manufactured notch filter (with 6 dB attenuator to improve input match)



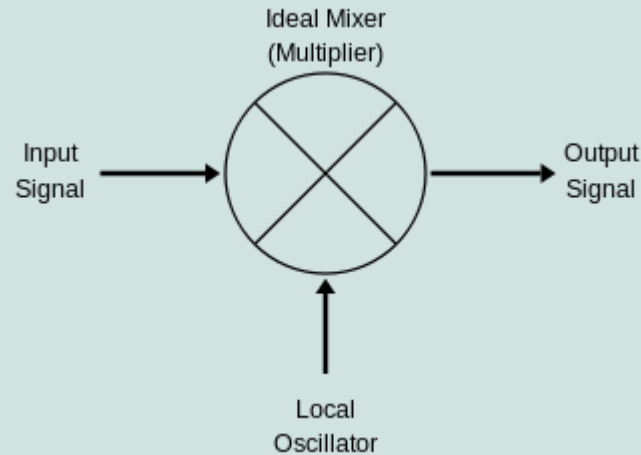
(d) Measured vs Simulated Responses

C. Copley

Notch Filter Applied to RFI



Hetrodyne Mixing



By trigonometric identity the multiplication of two sine waves is:

$$\sin(2\pi\nu_{\text{RF}}) \cdot \sin(2\pi\nu_{\text{LO}}) = \frac{1}{2} \cos(2\pi(\nu_{\text{RF}} - \nu_{\text{LO}})t) + \frac{1}{2} \cos(2\pi(\nu_{\text{RF}} + \nu_{\text{LO}})t)$$

Applying a low pass filter:

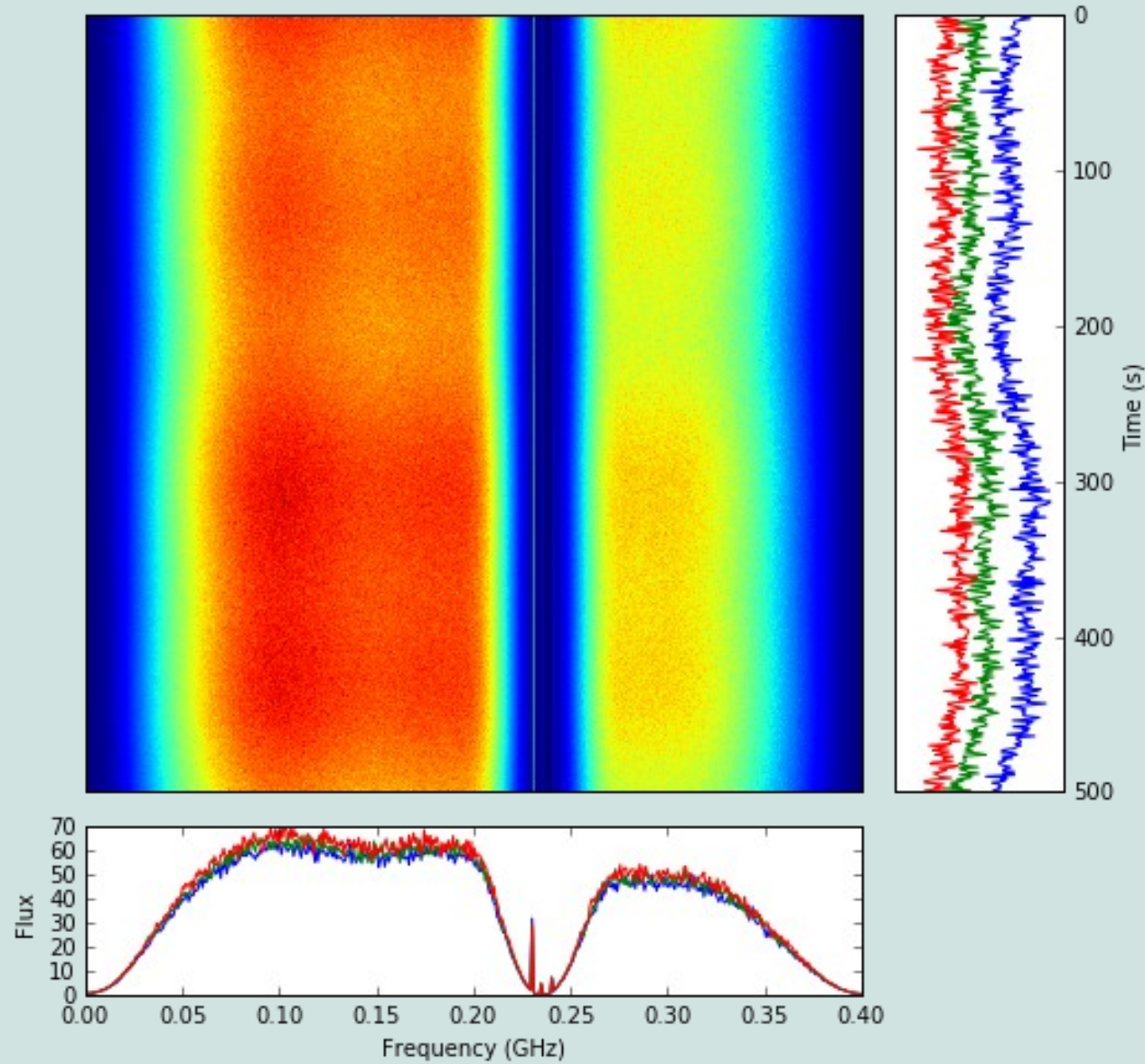
$$\sin(2\pi\nu_{\text{RF}}) \cdot \sin(2\pi\nu_{\text{LO}}) = \frac{1}{2} \cos(2\pi(\nu_{\text{RF}} - \nu_{\text{LO}})t) + \frac{1}{2} \cos(2\pi(\nu_{\text{RF}} + \nu_{\text{LO}})t)$$

The output frequency is:

$$\nu_{\text{IF}} \approx \frac{1}{2} \cos(2\pi(\nu_{\text{RF}} - \nu_{\text{LO}})t)$$

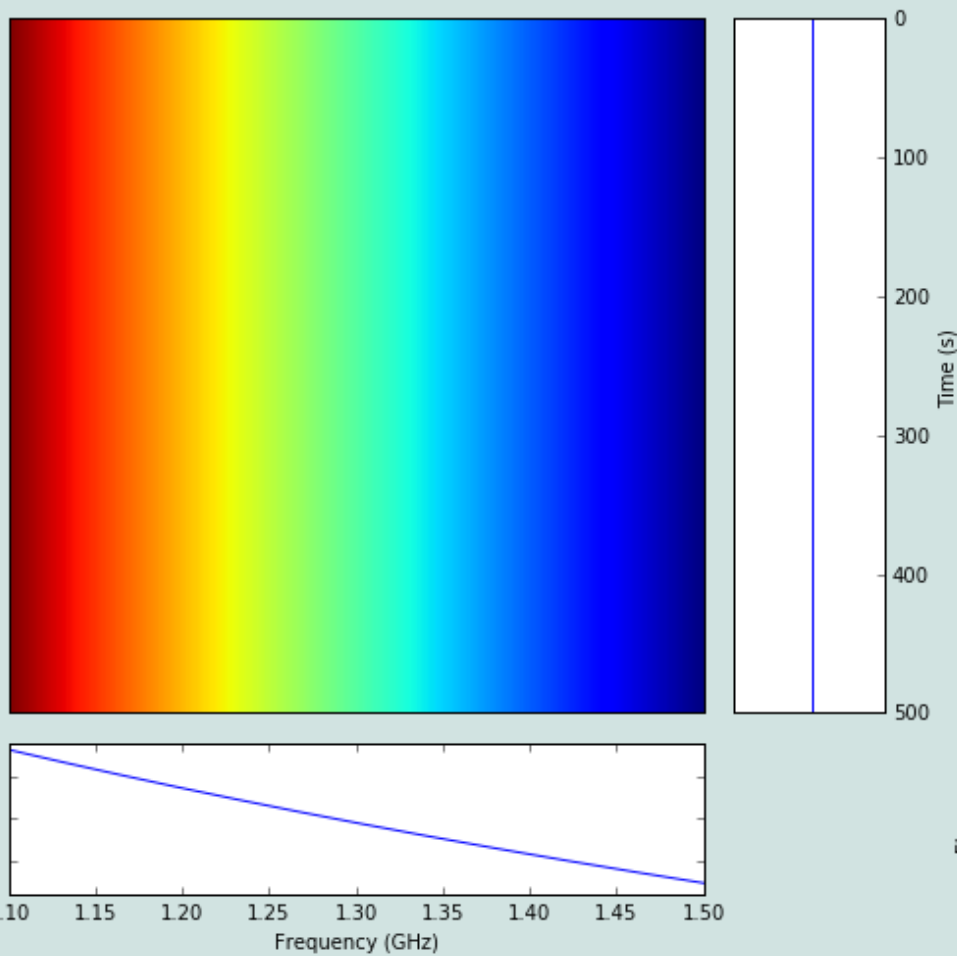
Add in System Noise

Observed Spectrum

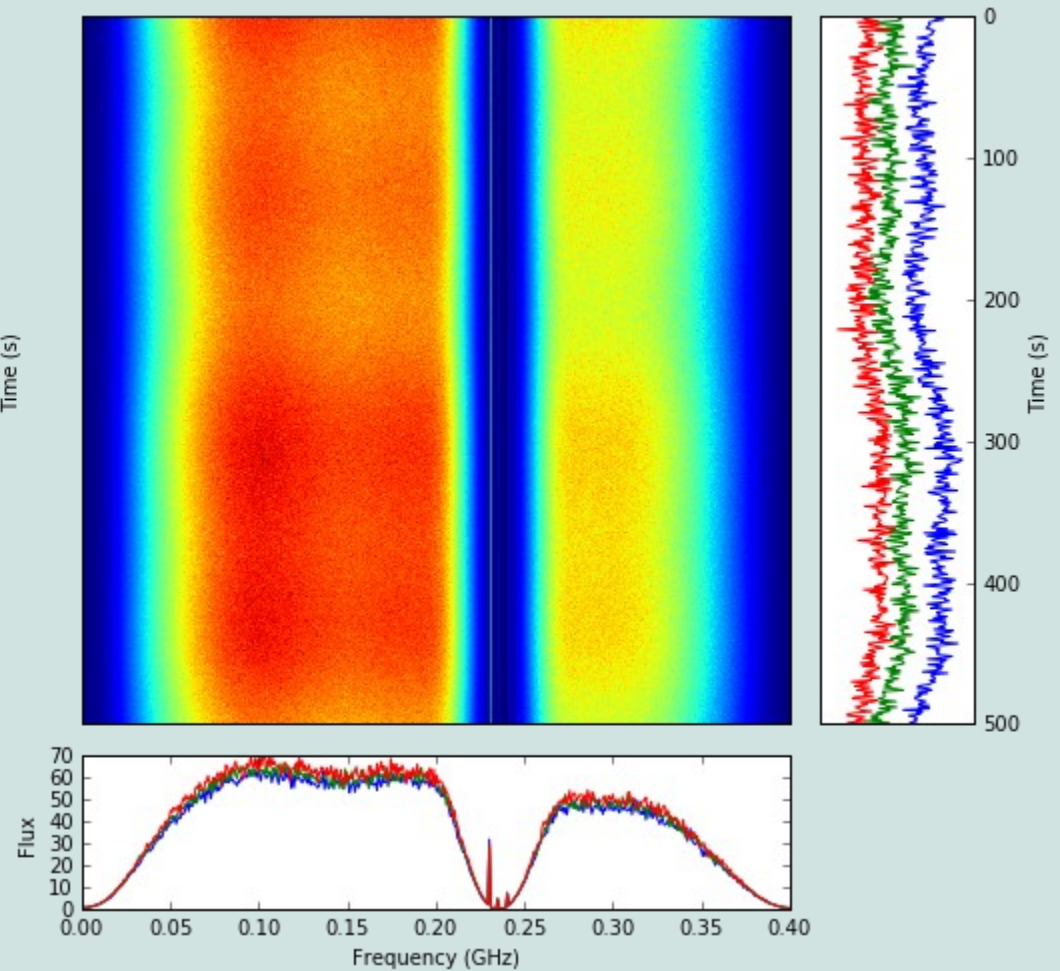


Analogue Response to an Ideal Source

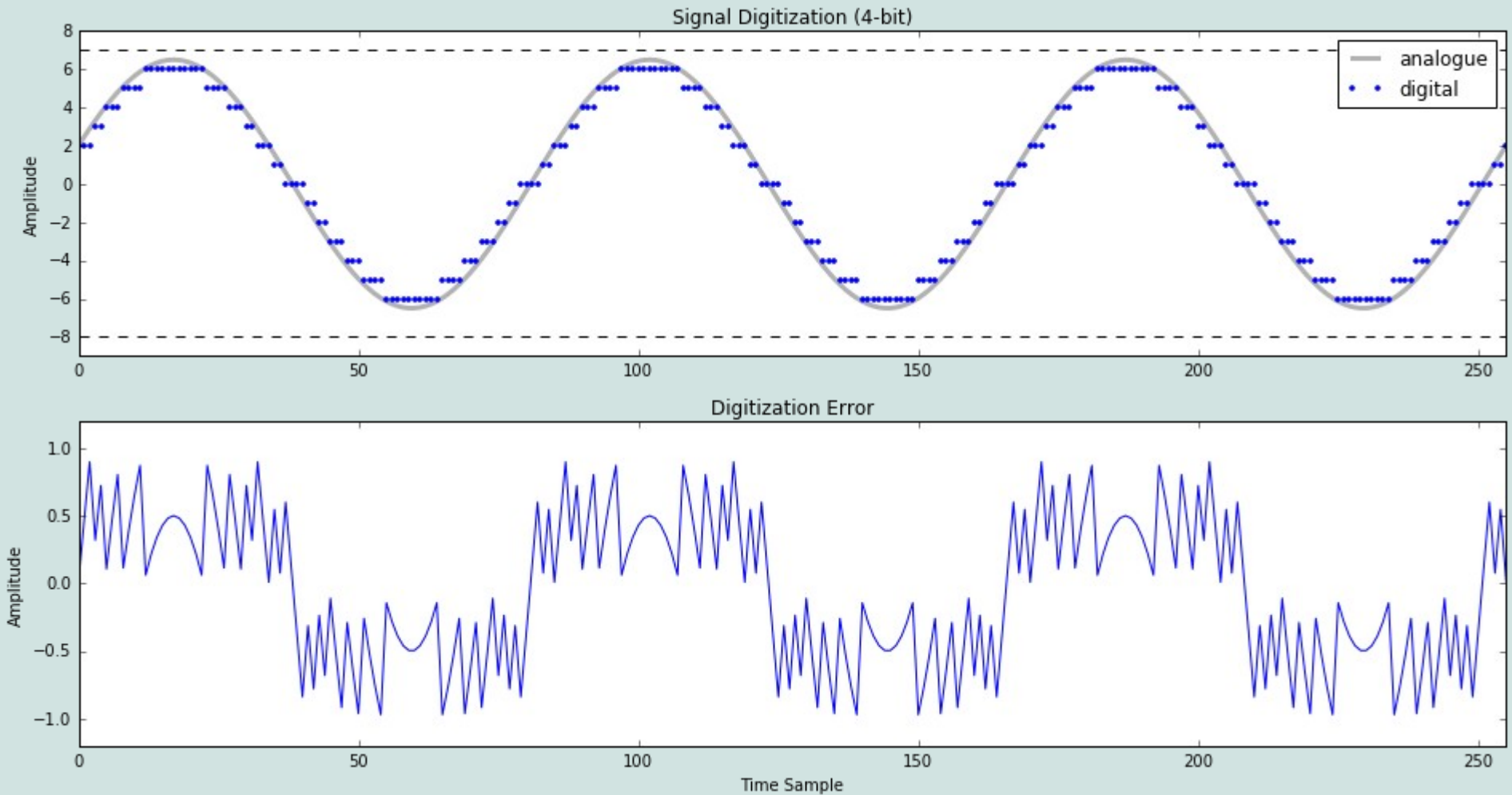
Idealized Source Spectrum



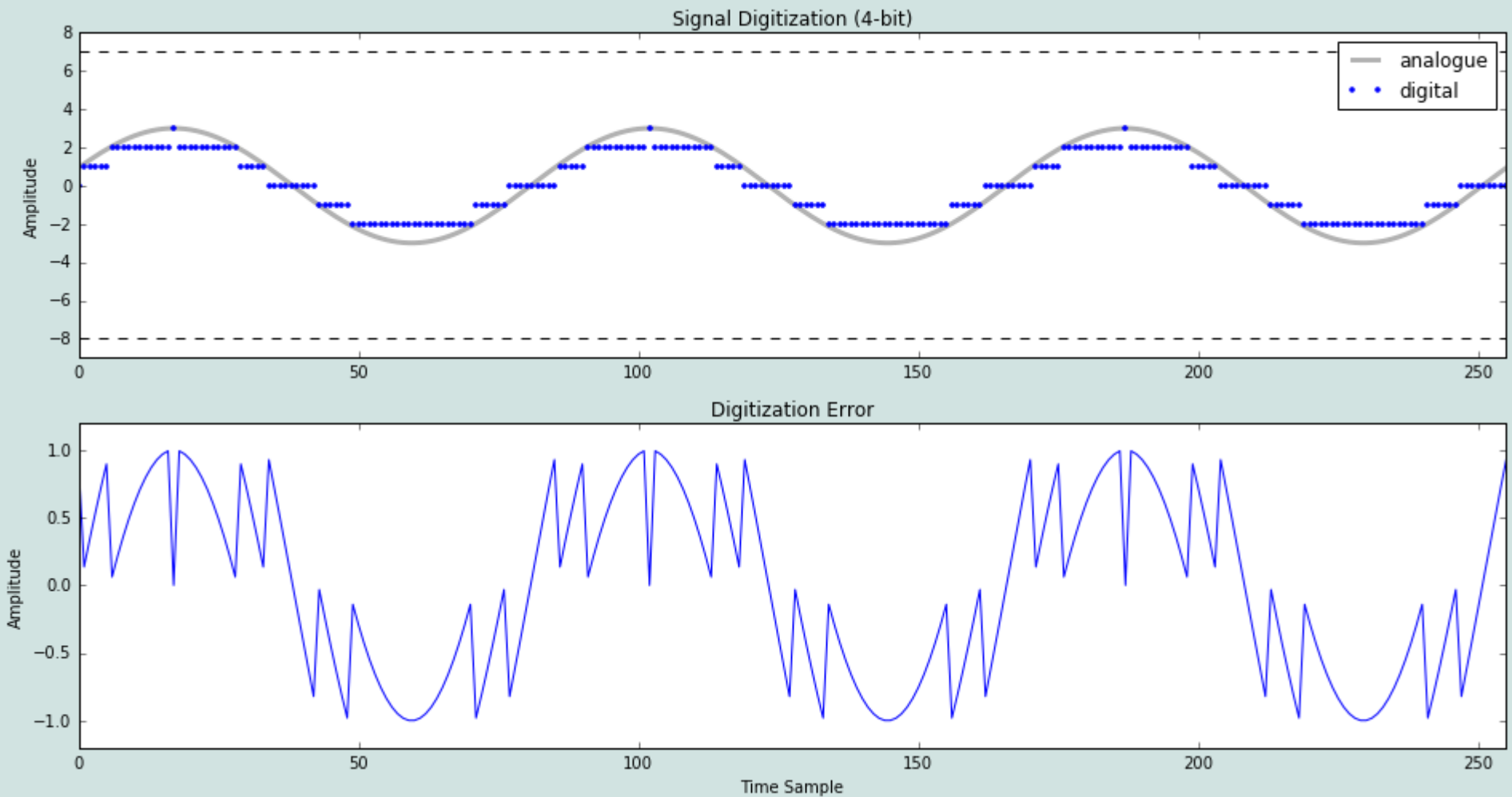
Observed Spectrum



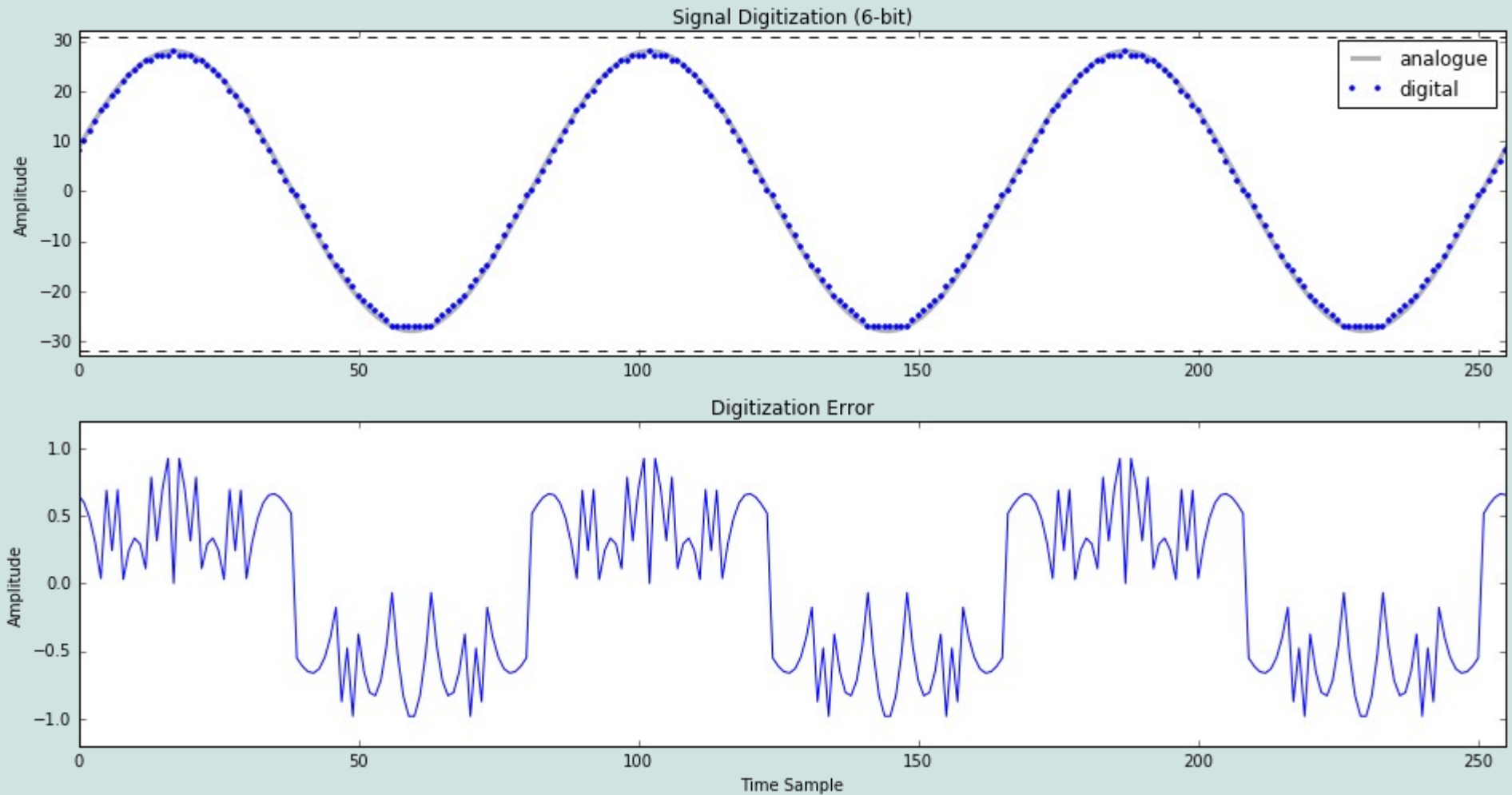
Digitization (Ideal Dynamic Range)



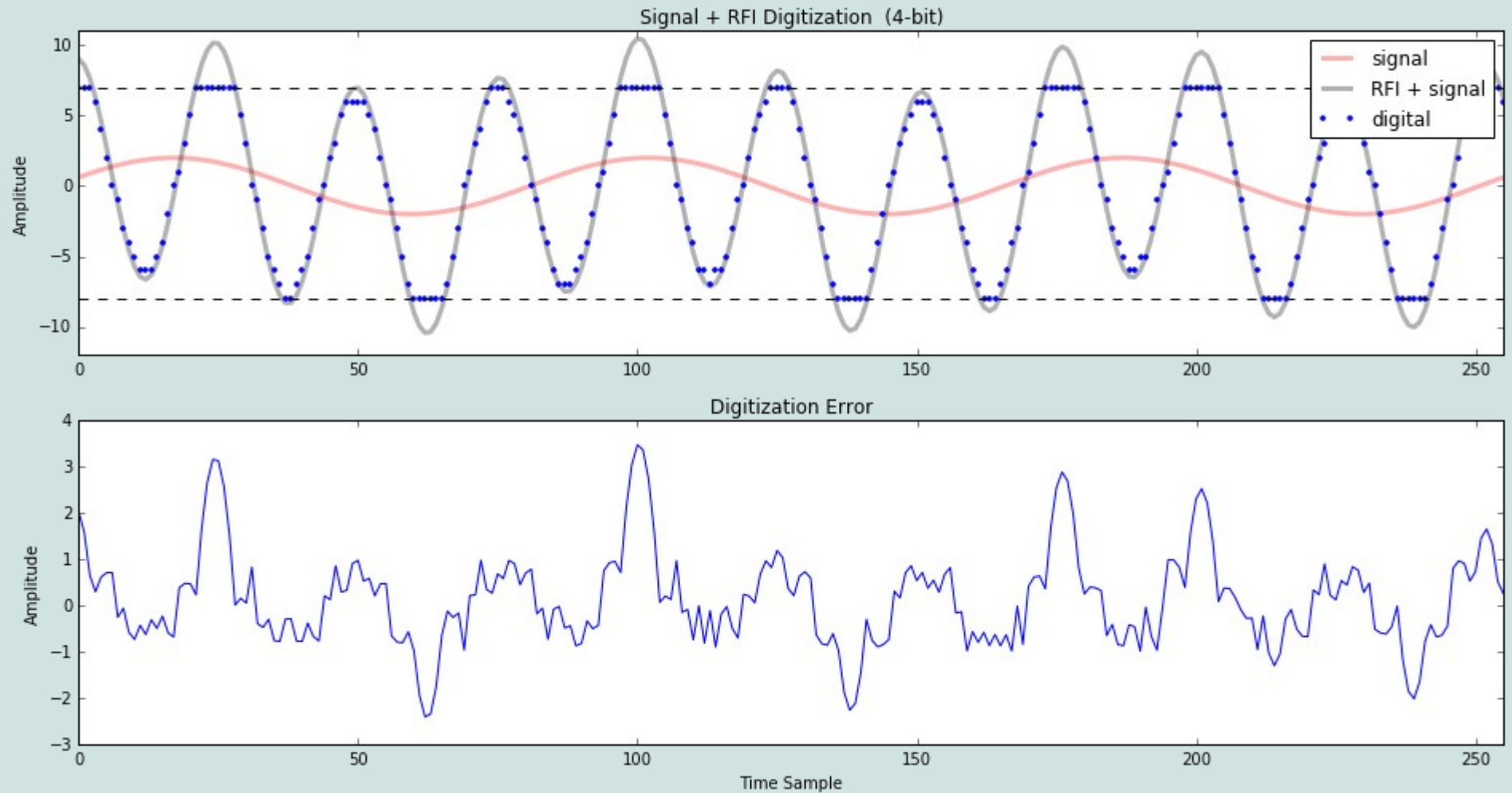
Digitization (Limited Dynamic Range)



Digitization (Increased Dynamic Range)



Digitization (Saturation)

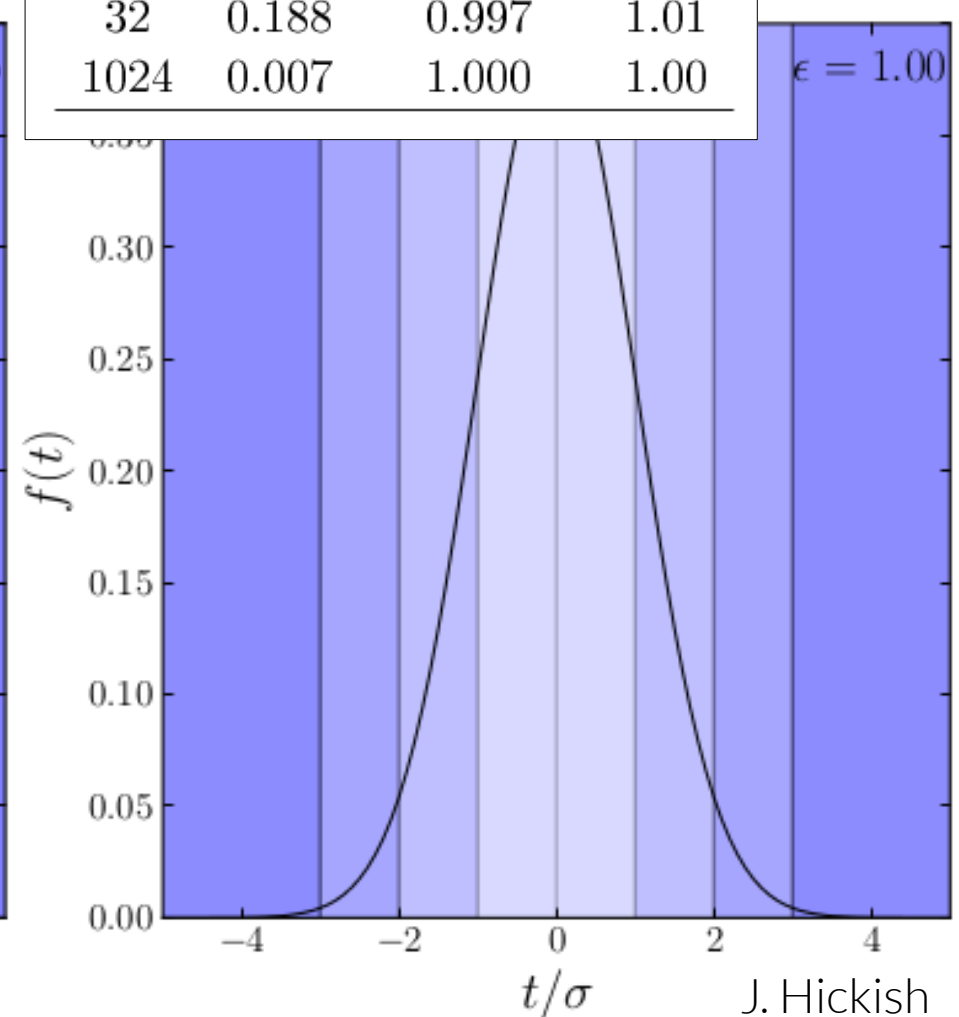
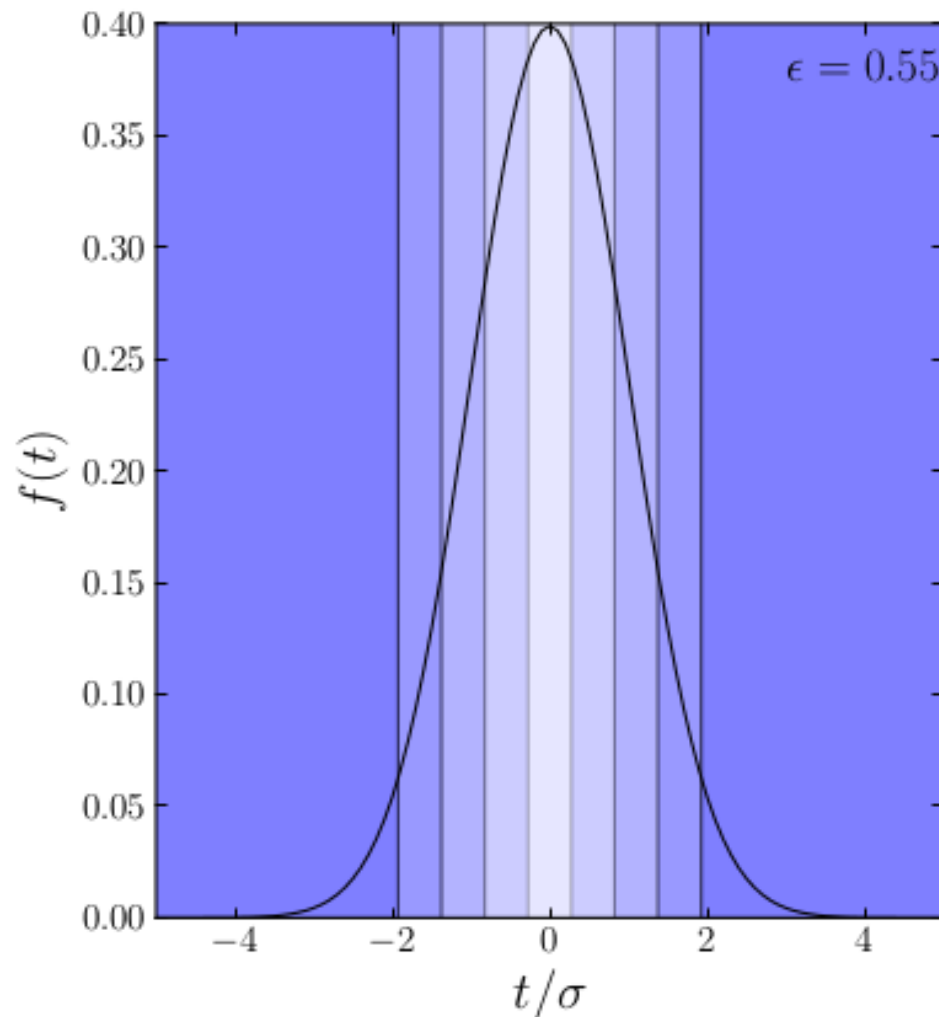


Quantization Efficiency

Low bit resolution quantization and efficiency

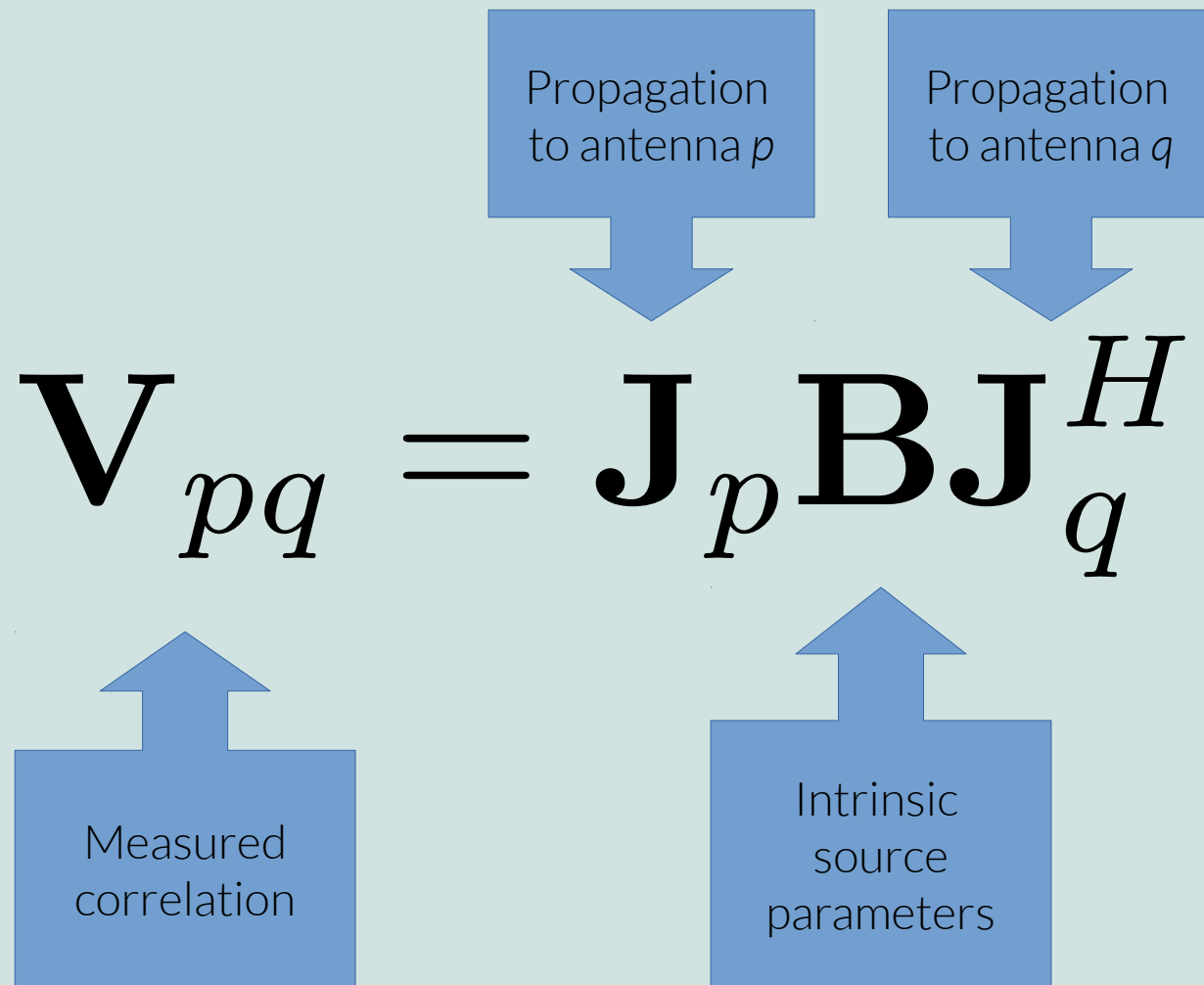
Key point: assuming a Gaussian signal (i.e. Astronomical signals), we only need 4 bits to get 98.9% efficiency

L	ϵ_{opt}	$\eta(L, \epsilon_{opt})$	Δt
2	-	0.637	2.46
3	0.612	0.810	1.52
7	0.674	0.950	1.11
8	0.575	0.966	1.07
15	0.358	0.987	1.03
16	0.334	0.989	1.02
31	0.194	0.996	1.01
32	0.188	0.997	1.01
1024	0.007	1.000	1.00

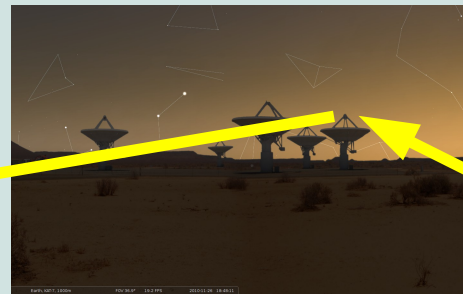
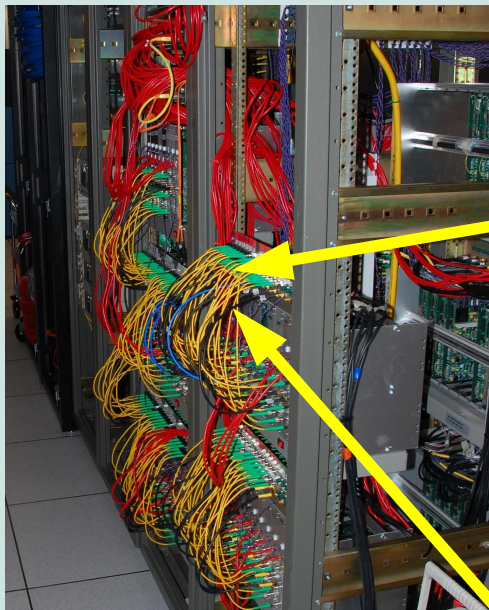


The Basic RIME

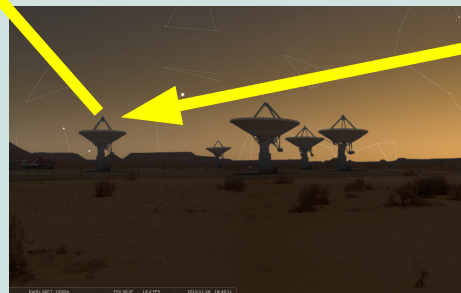
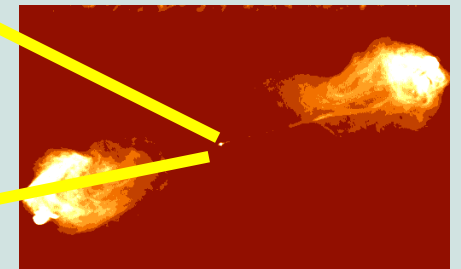
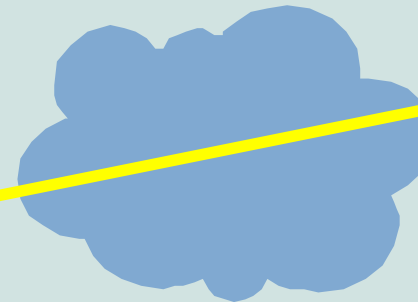
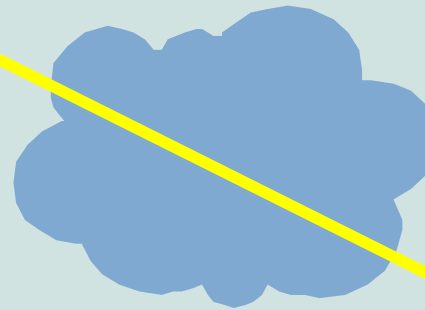
This gives us the basic form of the RIME:



Correlation



$$\mathbf{v}_p = \mathbf{J}_p \mathbf{e}$$



$$\mathbf{v}_q = \mathbf{J}_q \mathbf{e}$$

$$\begin{aligned} V_{11} &= \langle v_{p1} v_{q1}^* \rangle \\ V_{22} &= \langle v_{p2} v_{q2}^* \rangle \\ V_{12} &= \langle v_{p1} v_{q2}^* \rangle \\ V_{21} &= \langle v_{p2} v_{q1}^* \rangle \end{aligned}$$

A correlator computes four complex pairwise products called *correlations*.

Convolution Theorem and Correlation

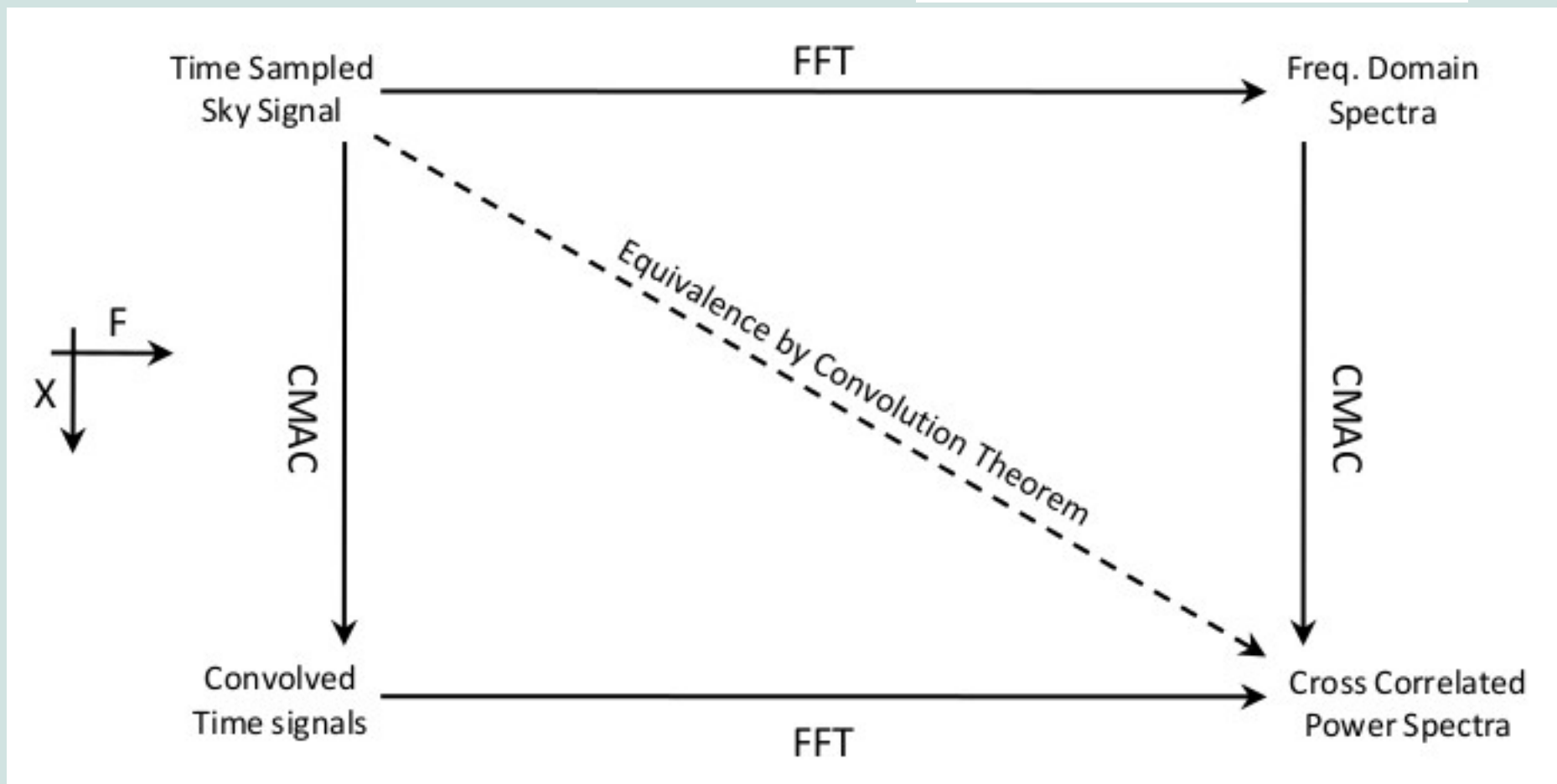
To compute visibilities, we would like to correlate (convolve) for each antenna pair (f,g)

Convolution Theorem:

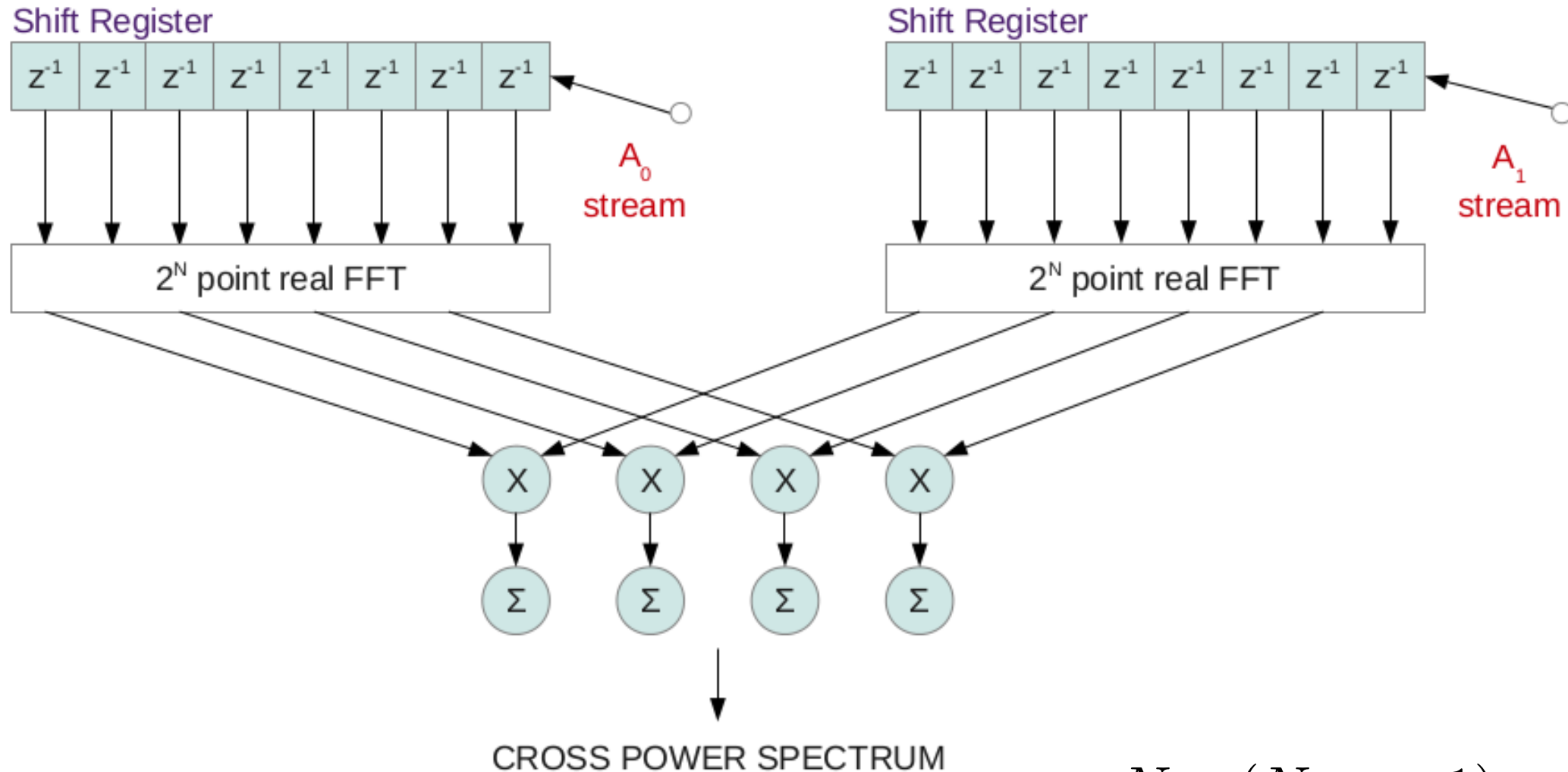
$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \mathcal{F}\{g\}$$

Where the convolution symbol is defined as:

$$f * g = \int f(x)g(z - x)dx$$



FX Correlator

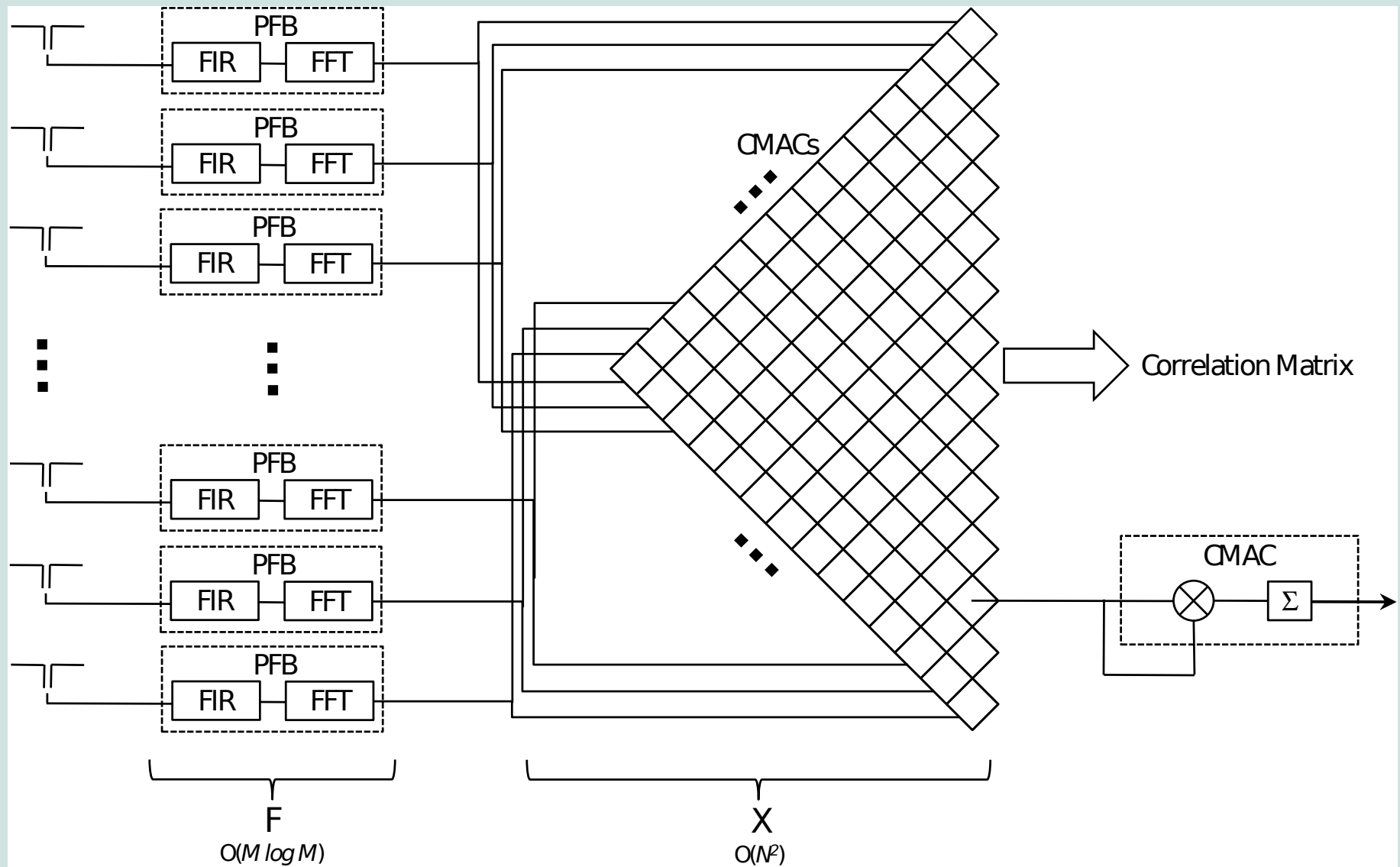


Cost: $O(NM \log(M) + MN^2)$
 M: frequency channels
 N: number of antennas

$$N_{bls} = N_{ant} + \frac{N_{ant}(N_{ant} - 1)}{2}$$

Auto-correlations \nearrow \nwarrow Cross-correlations

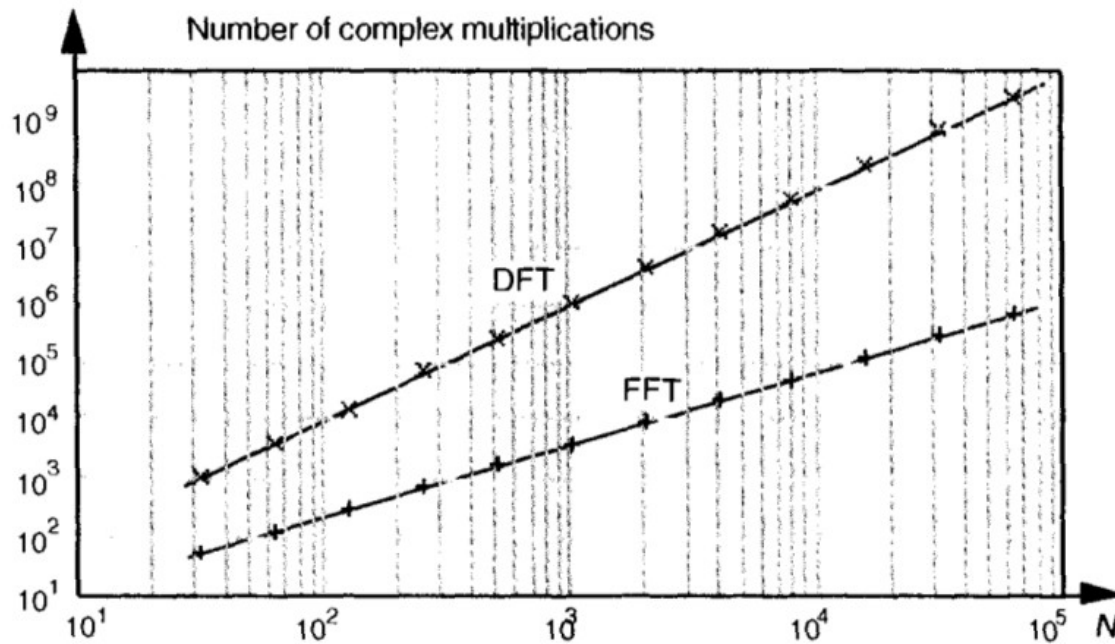
FX Correlator



Fast Fourier Transform (FFT)

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{i2\pi}{N}nk}$$

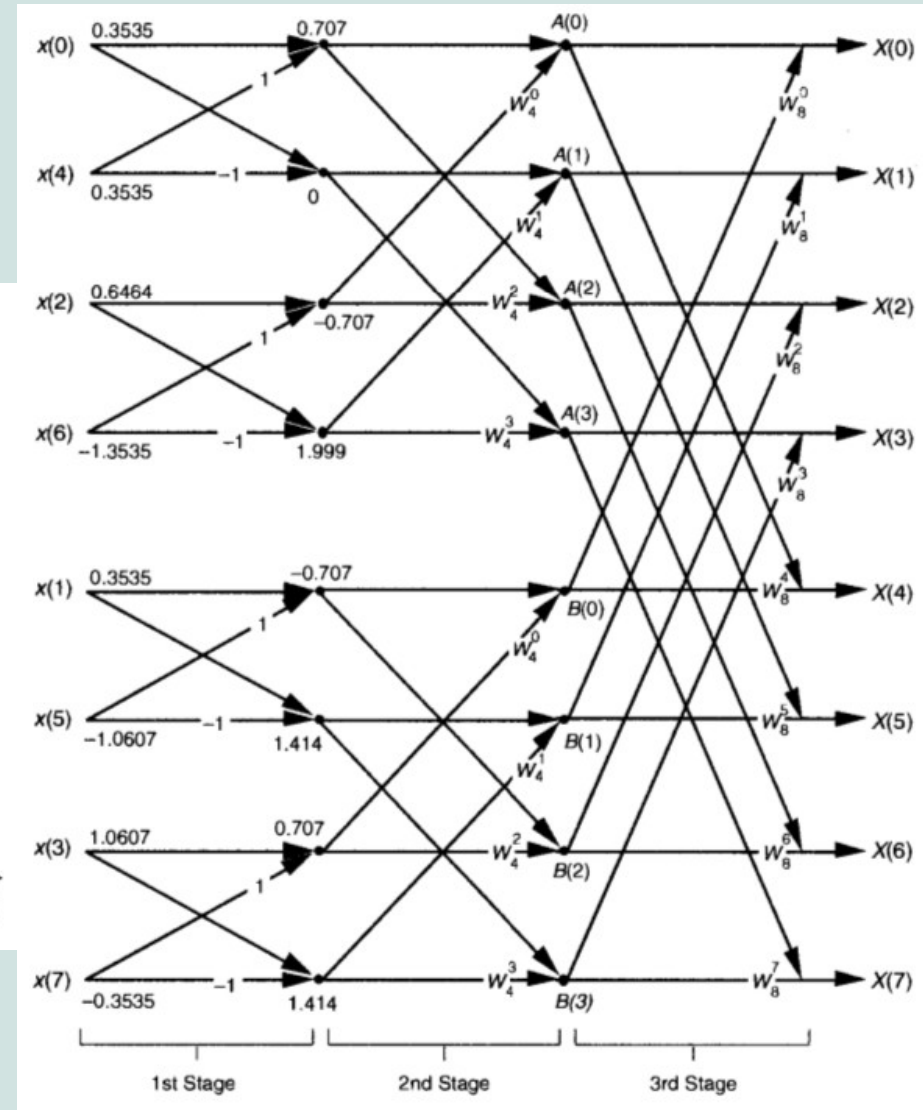
Discrete Fourier Transform (DFT)



DFT Cost: $O(N^2)$

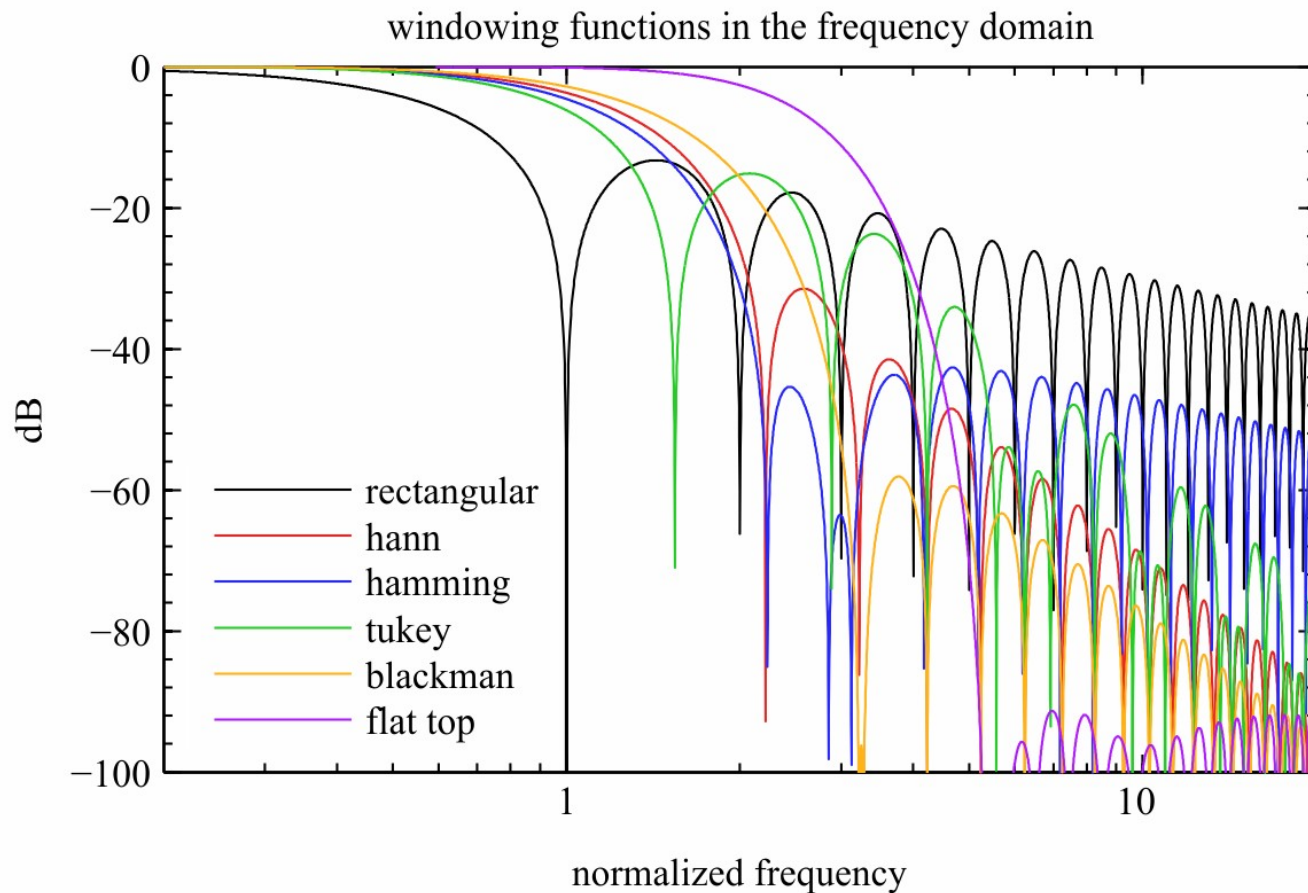
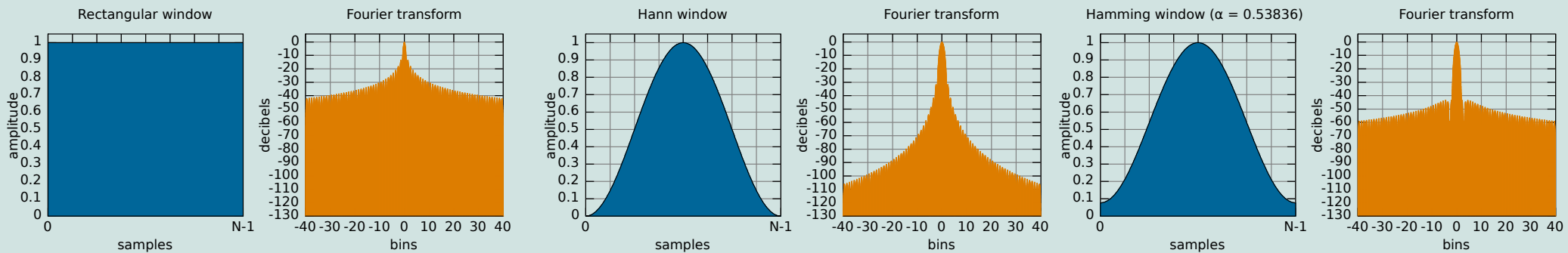
FFT Cost: $O(N \log N)$

Radix-2 DIT 8-point
Cooley-Tukey FFT

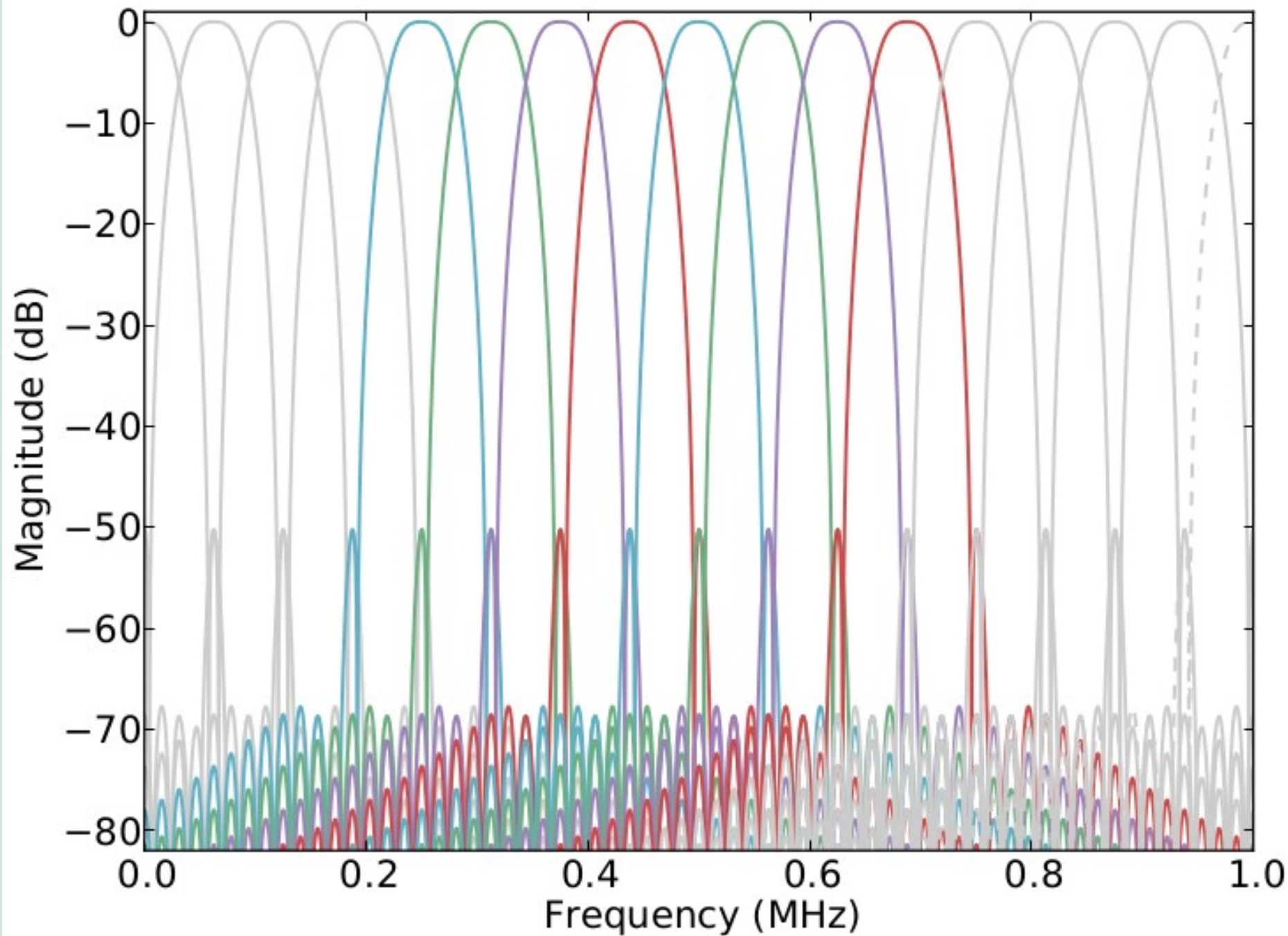


Almost all FFT implementations use a radix-2 system, so FFT of size 2^N are ideal. Try to avoid Fourier transforms of prime number size.

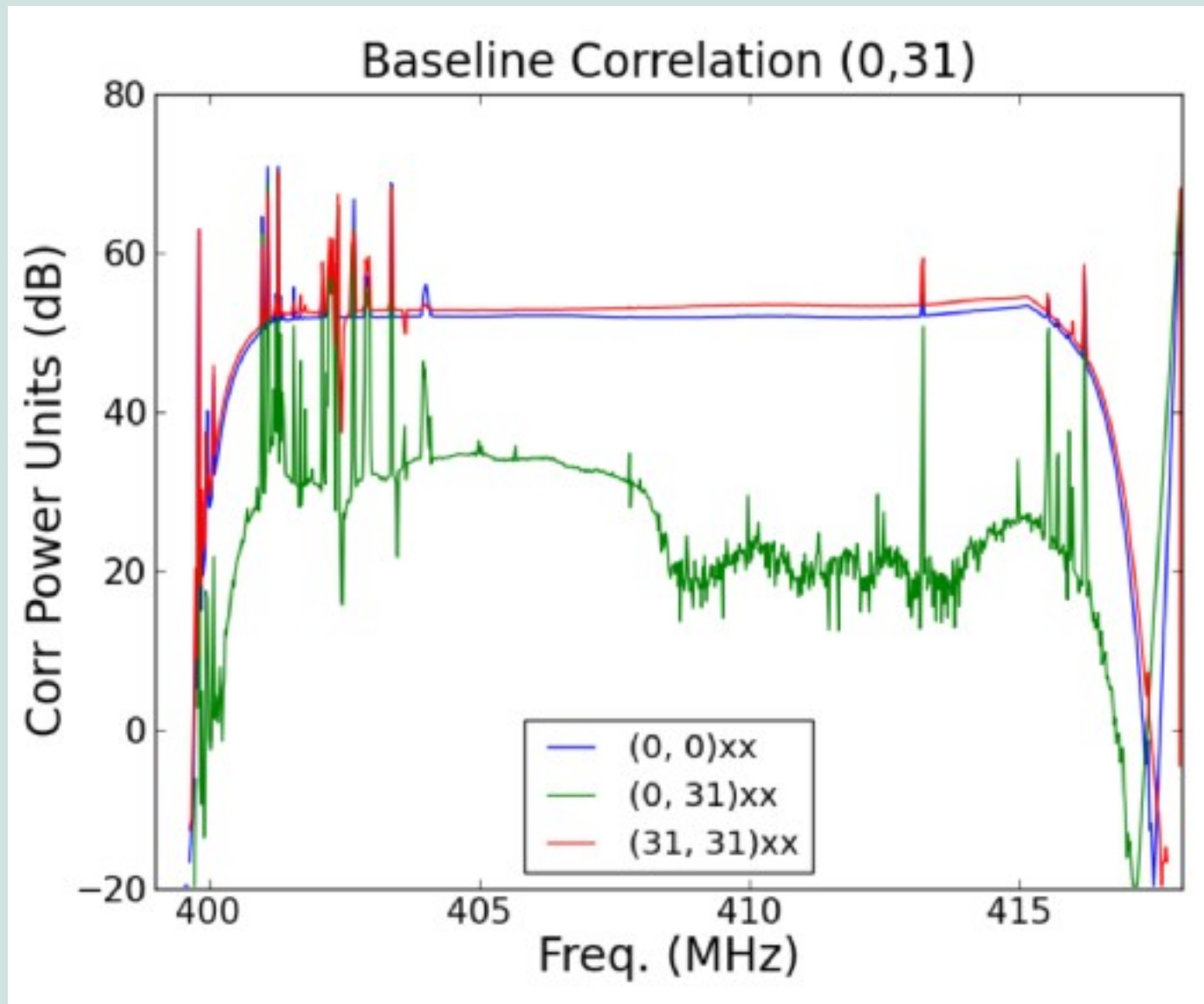
Finite Impulse Response (FIR) Window Functions

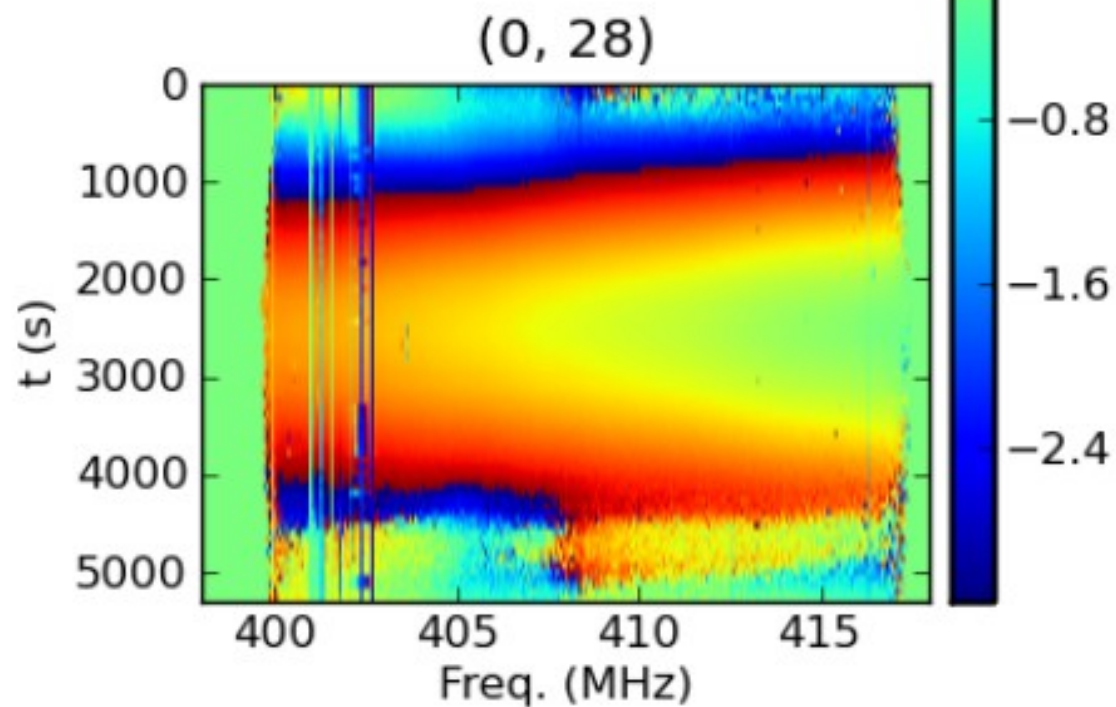
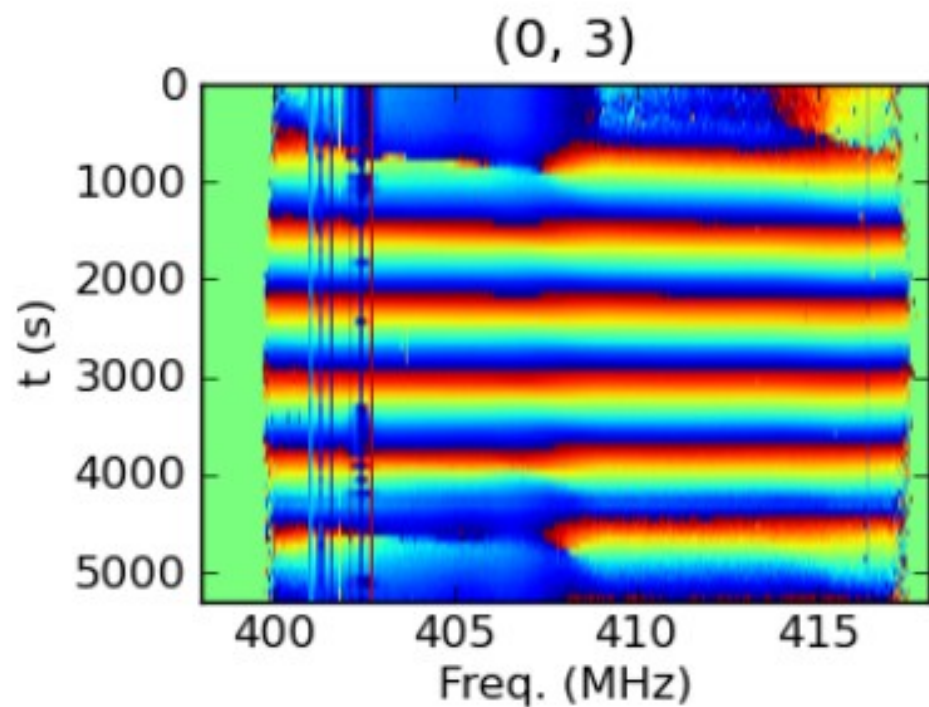
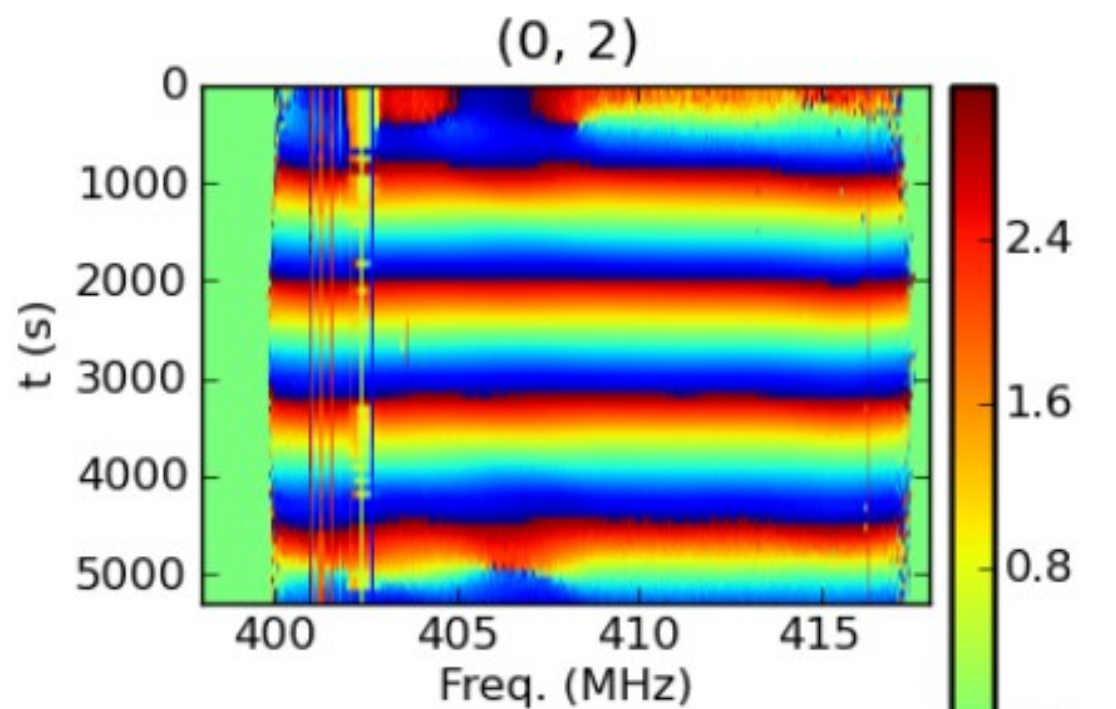
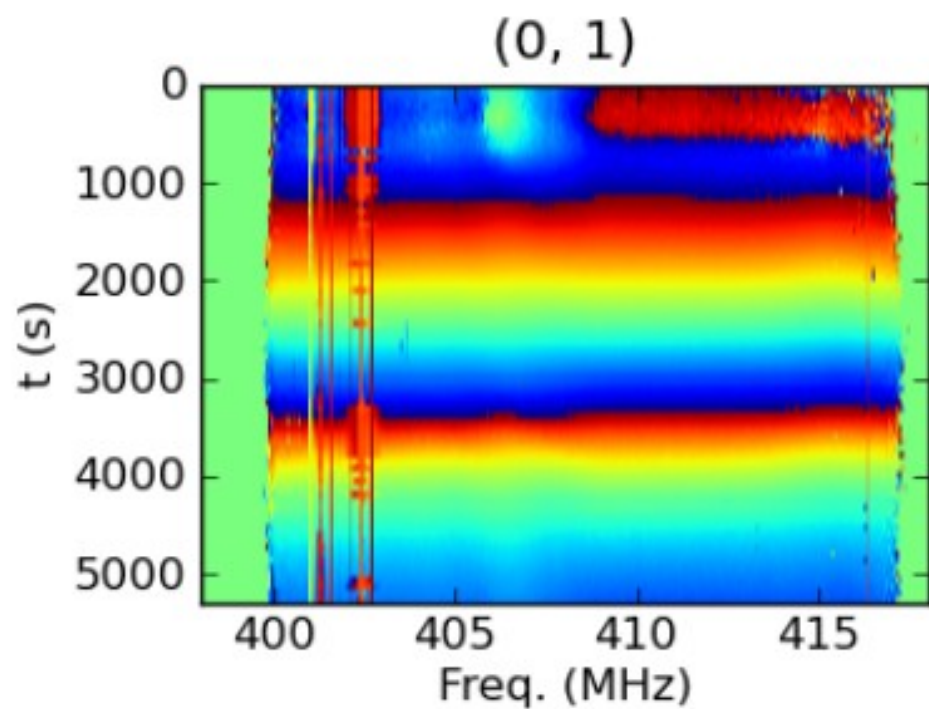


Polyphase Filter Banks (PFBs)



Baseline Spectrum





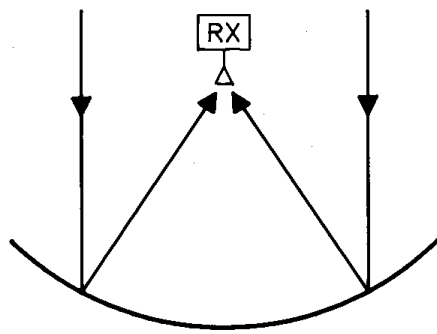
Primary Beam (E-Jones)

$$\mathbf{E}(\theta, \phi, \nu) = \begin{pmatrix} E_{l \rightarrow l}(\theta, \phi, \nu) & E_{l \rightarrow r}(\theta, \phi, \nu) \\ E_{r \rightarrow l}(\theta, \phi, \nu) & E_{r \rightarrow r}(\theta, \phi, \nu) \end{pmatrix}$$

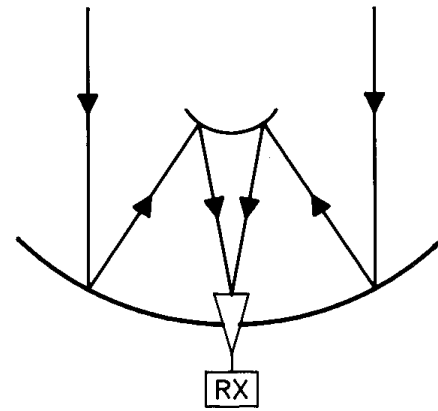
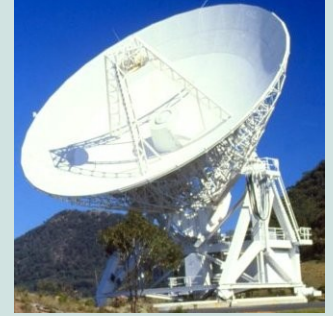
The *position*- and *frequency*-dependent effect of the physical structure.

Potentially also *time*-dependent in the case of an Altitude-Azimuth mount.

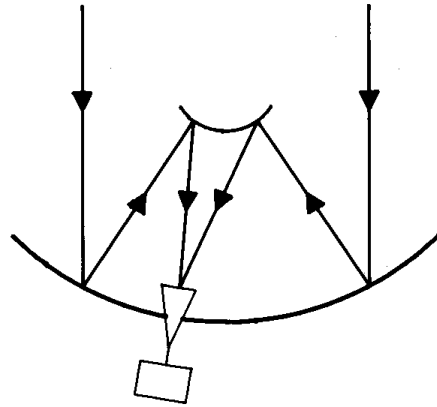
Prime Focus (GMRT)



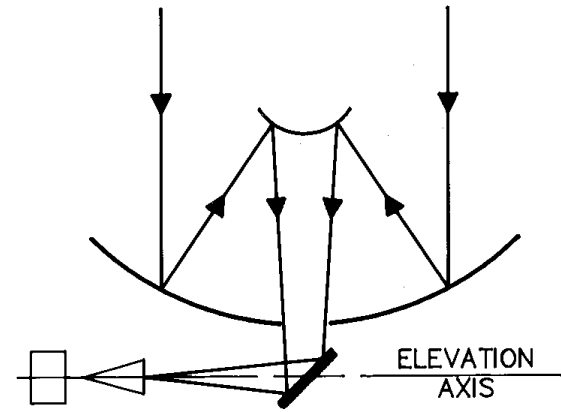
Cassegrain (ATCA)



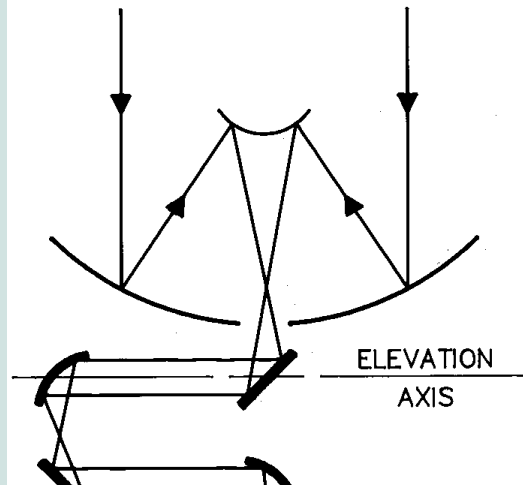
Offset Cassegrain (VLA)



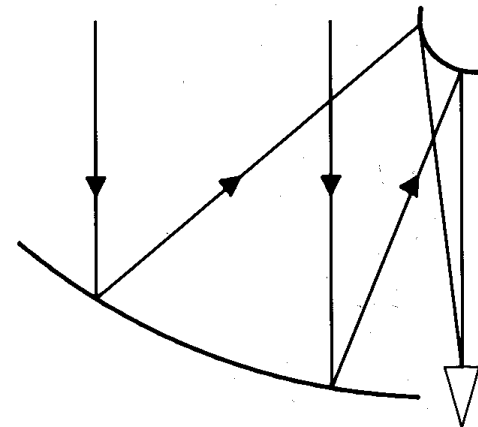
Nasmyth (CARMA)



Bent Nasmyth (SMA)



Offset Gregorian (GBT)



Aperture Efficiency

$$\eta = \eta_{\text{surface}} \eta_{\text{blockage}} \eta_{\text{spillover}} \eta_{\text{taper}} \dots$$

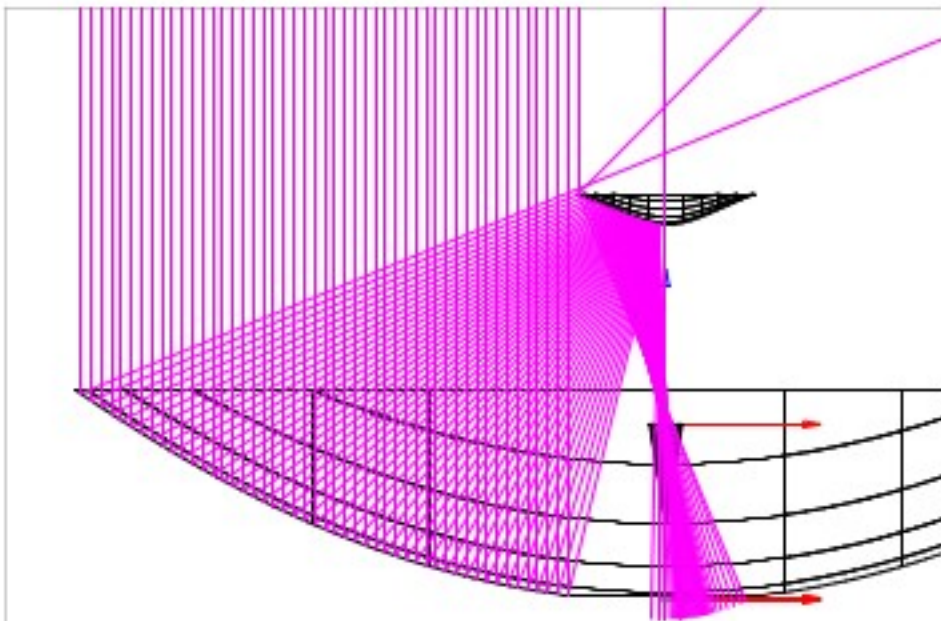
η_{surface} : any surface has reflective loss

η_{blockage} : the structure above the dish block a portion of the light (to 0th order)

$\eta_{\text{spillover}}$: loss due to the caustic illumination onto the receiver feed

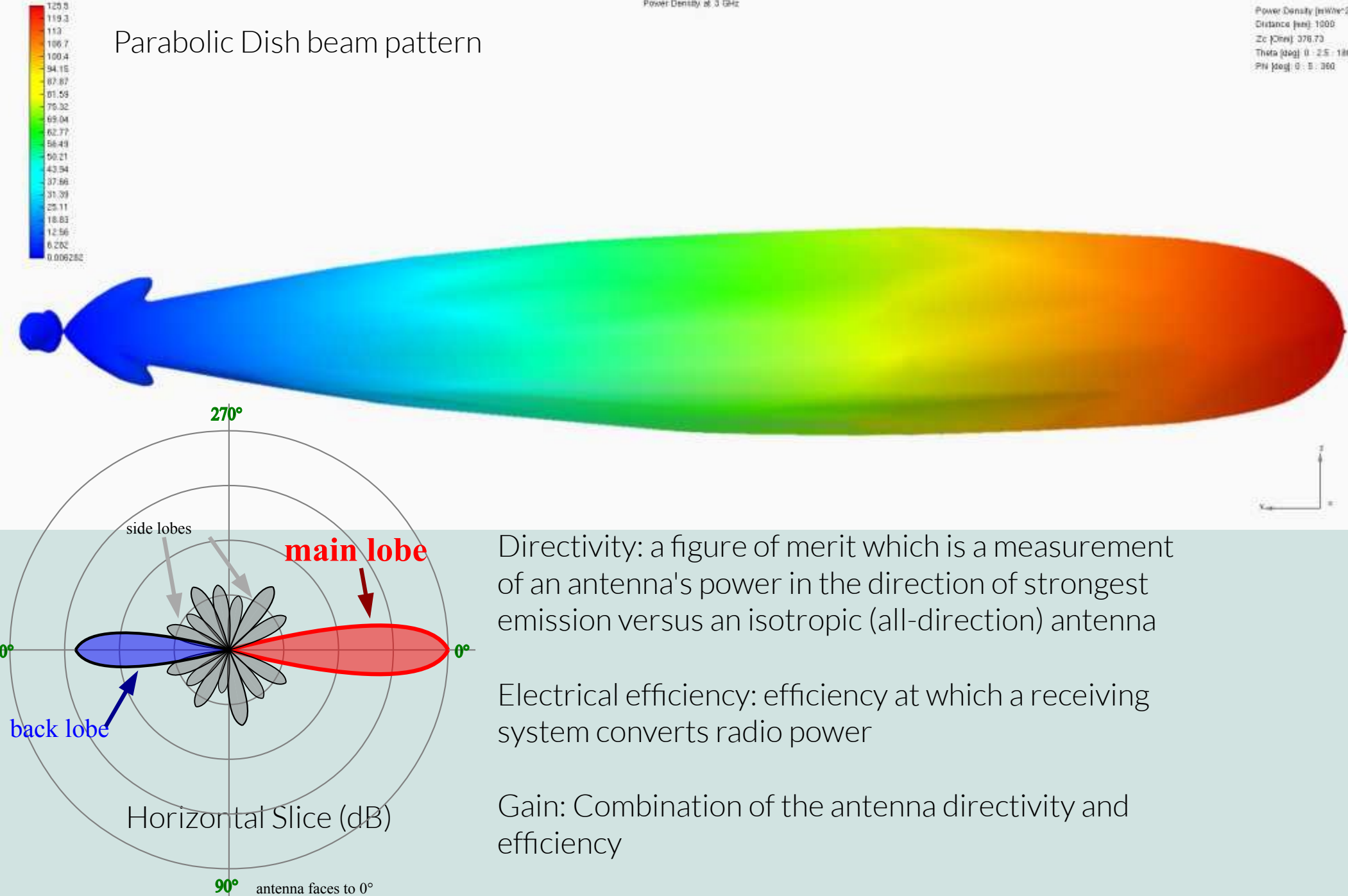
η_{taper} : there is a radius dependent loss with respect to illumination

These efficiencies are approximate metrics, in reality, a electro-magnetic model of the primary beam provides a more complete description



C. Copley

Parabolic Dish beam pattern



Directivity: a figure of merit which is a measurement of an antenna's power in the direction of strongest emission versus an isotropic (all-direction) antenna

Electrical efficiency: efficiency at which a receiving system converts radio power

Gain: Combination of the antenna directivity and efficiency

Phased Array Antenna Handbook : Mailloux



Receivers (D- and C-Jones)

Leakage between orthogonal feeds:

$$\mathbf{D}(\theta, \phi, \nu) = \begin{pmatrix} 1 & d(\theta, \phi, \nu) \\ d(\theta, \phi, \nu) & 1 \end{pmatrix}$$
$$d \ll 1$$

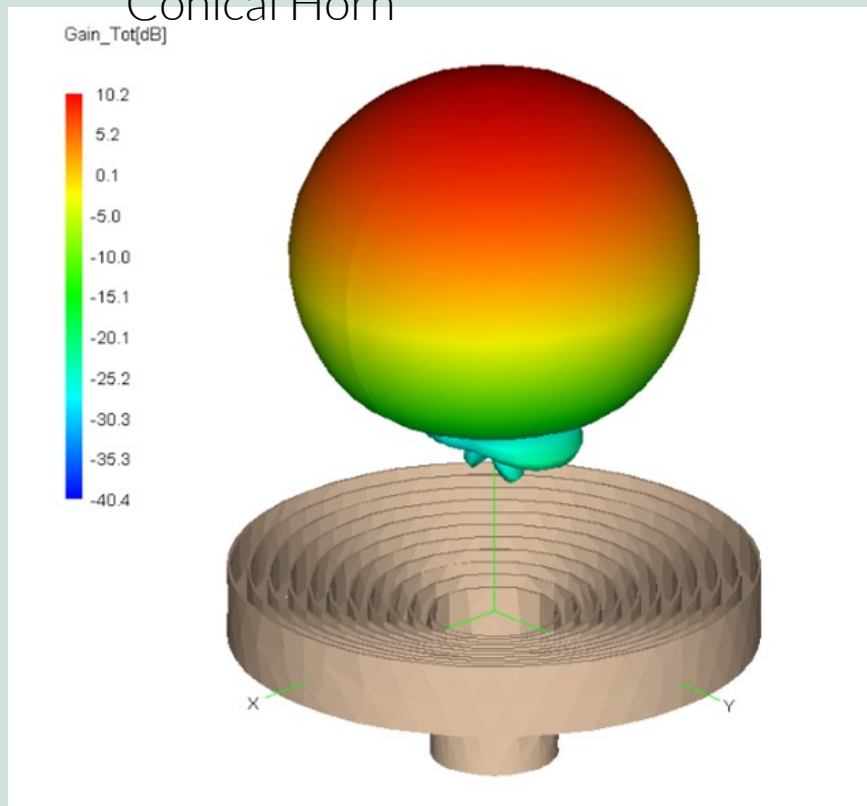
Configuration matrix to convert between reference frames, such as linear to circular:

$$\mathbf{C}_{\text{lin} \leftrightarrow \text{circ}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

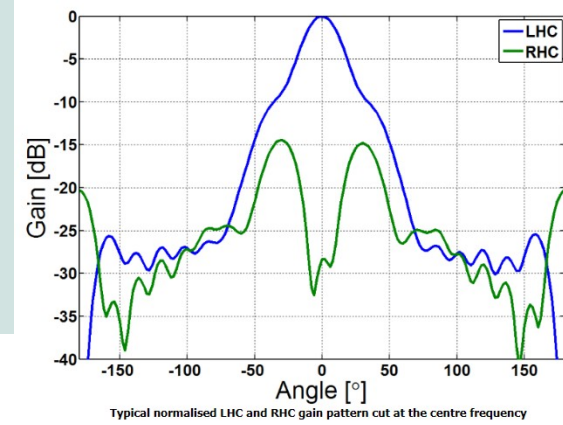
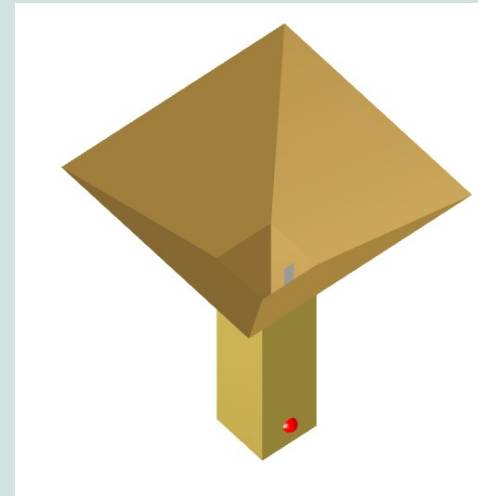
Receivers (D- and C-Jones)



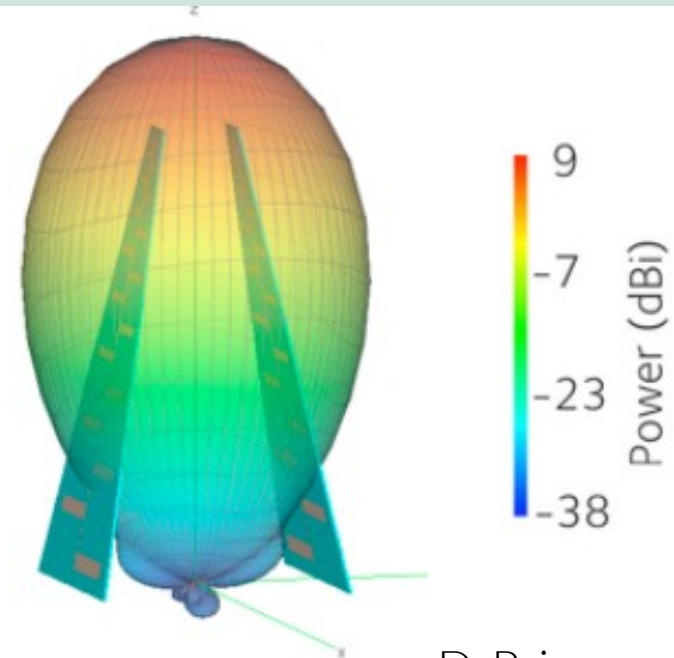
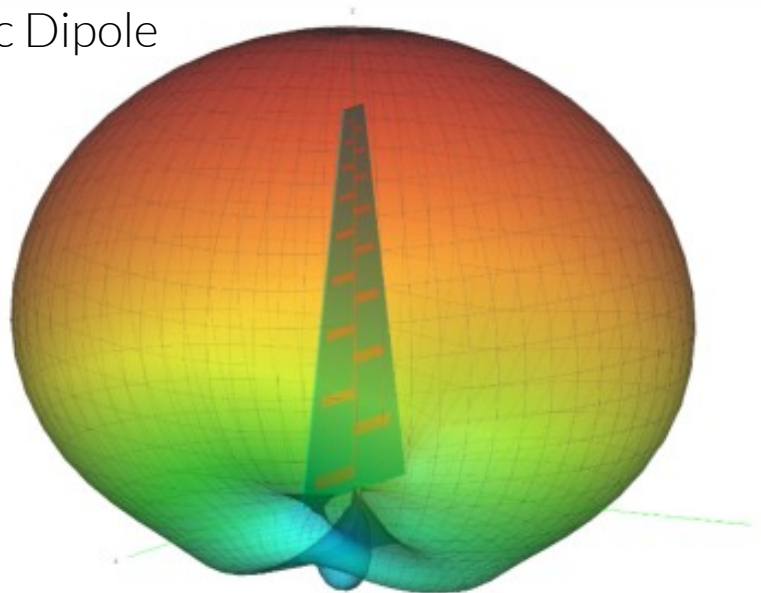
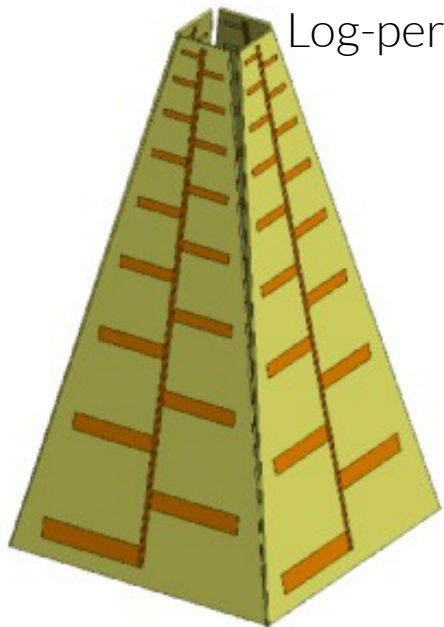
Conical Horn



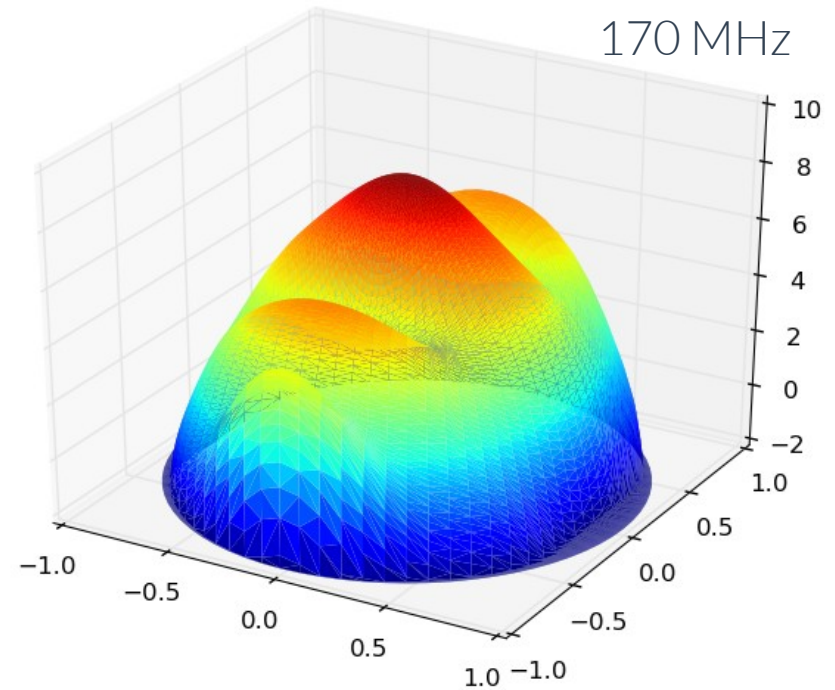
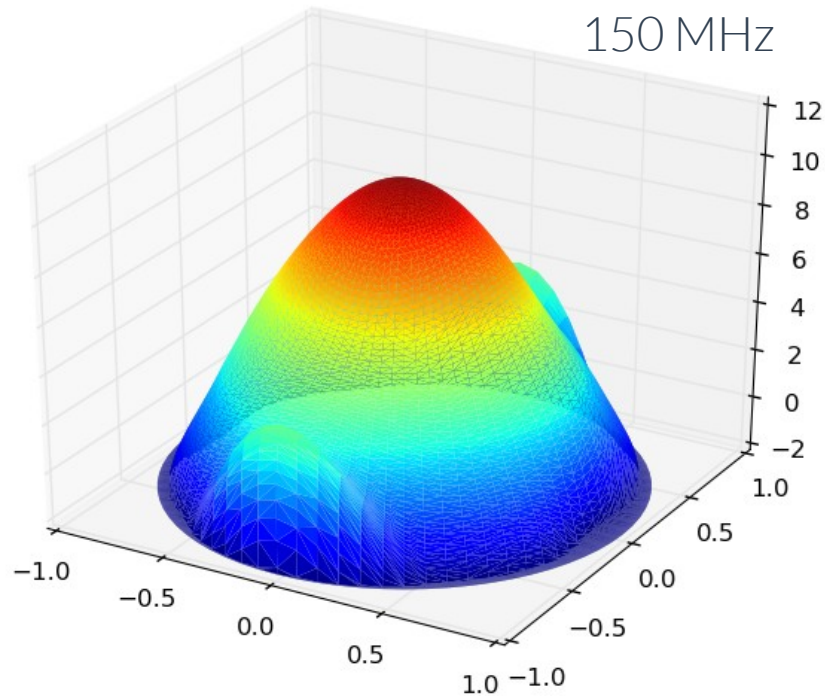
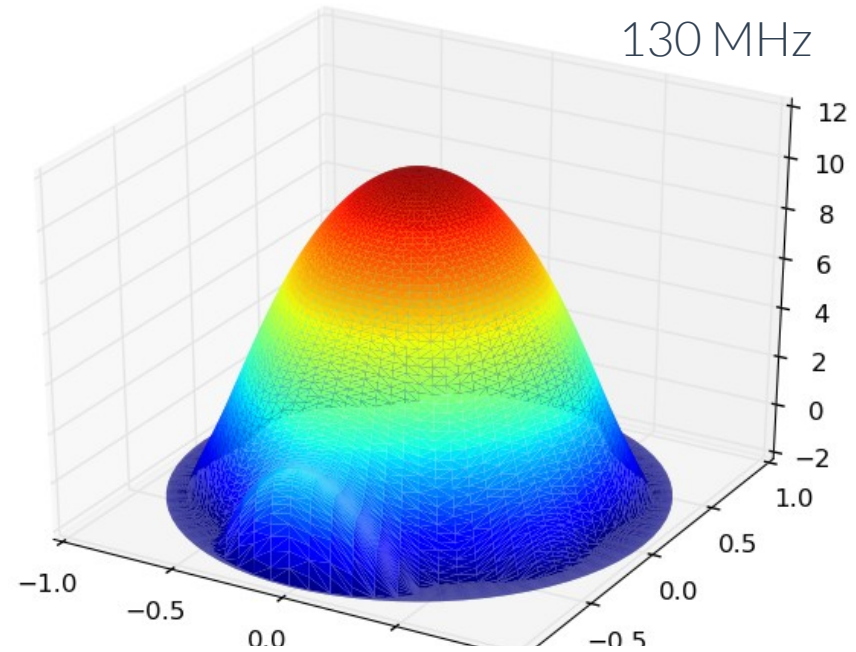
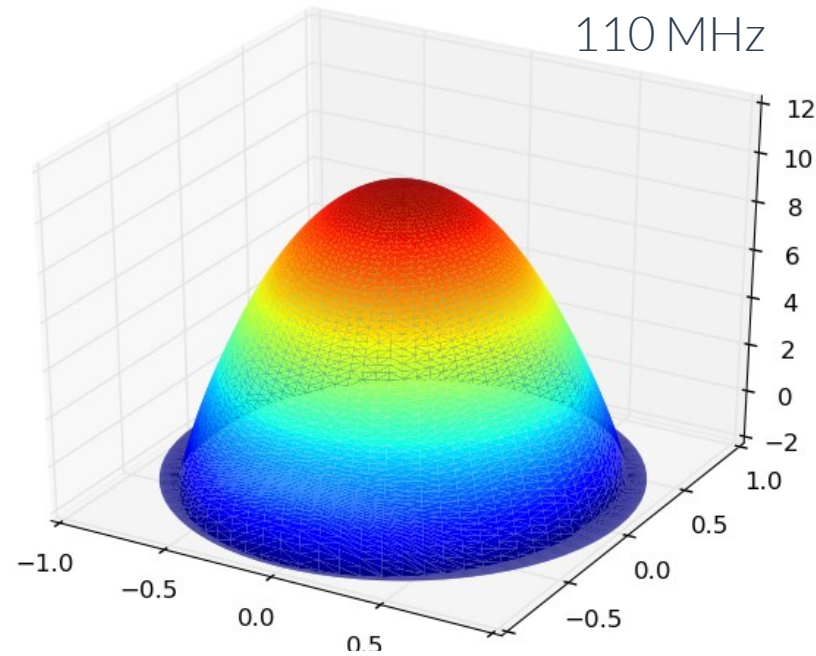
Square Horn

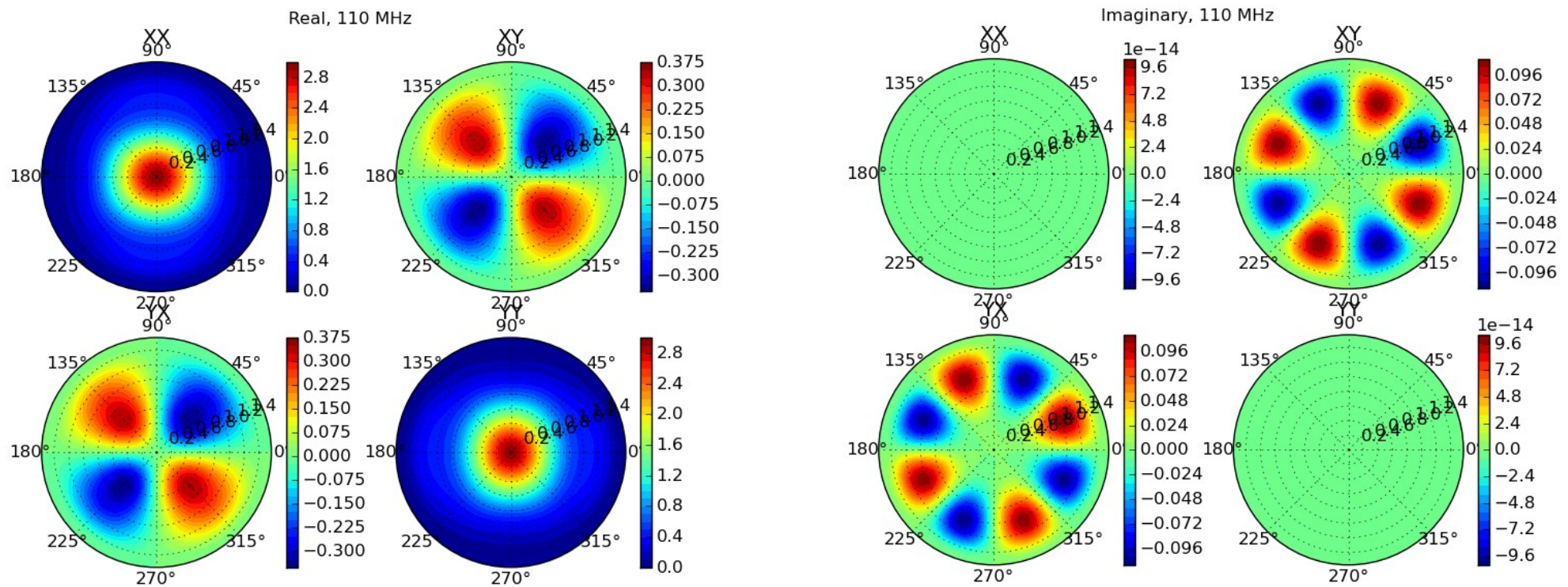


Log-periodic Dipole

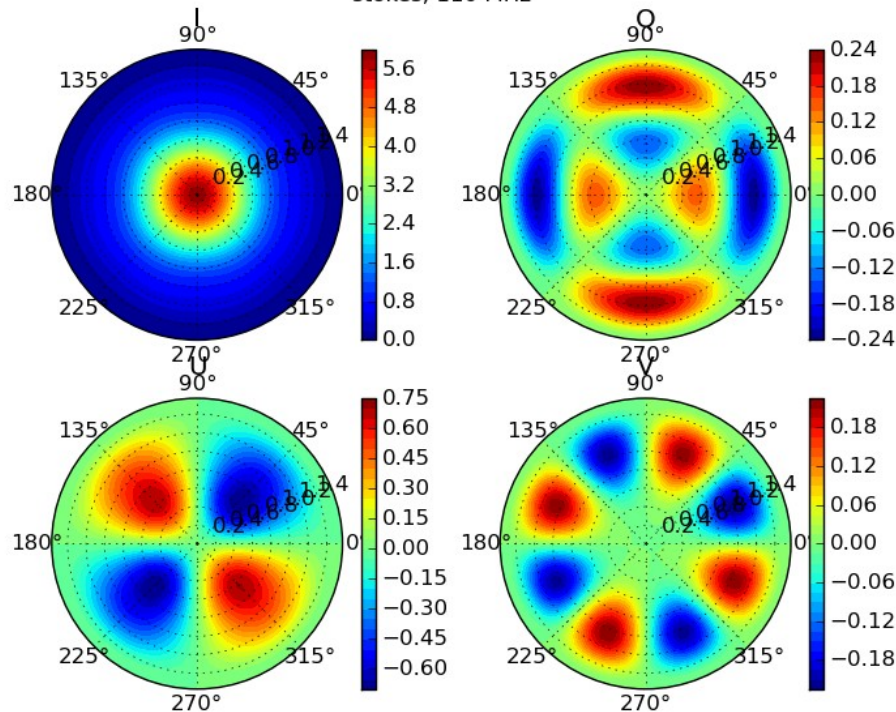


Receiver Frequency Dependence





Jones representation conversion to Stokes Parameters



$$\mathbf{M}_{\mathbf{E}} = \mathbf{S}^{-1} (\mathbf{E} \otimes \mathbf{E}^*) \mathbf{S}$$

Measurement in Basis Set

If a source is circularly polarized, there is no signal loss using an orthogonal linear feed system. And the same for a linearly polarized source and circular feeds system.

So, ideally, if we are measuring a source with a particular polarization we would use the other polarization type as the receiver feed. But, in reality certain feed types are desirable for different designs.

Conversion between linear and circular basis is done via a *quarter wave plate*.

$$\mathbf{C}_{\text{lin} \leftrightarrow \text{circ}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

Polarization Leakage (D-Jones)

Intrinsic Cross-Polarization Ratio (IXR) [Carozzi & Woan 2011]

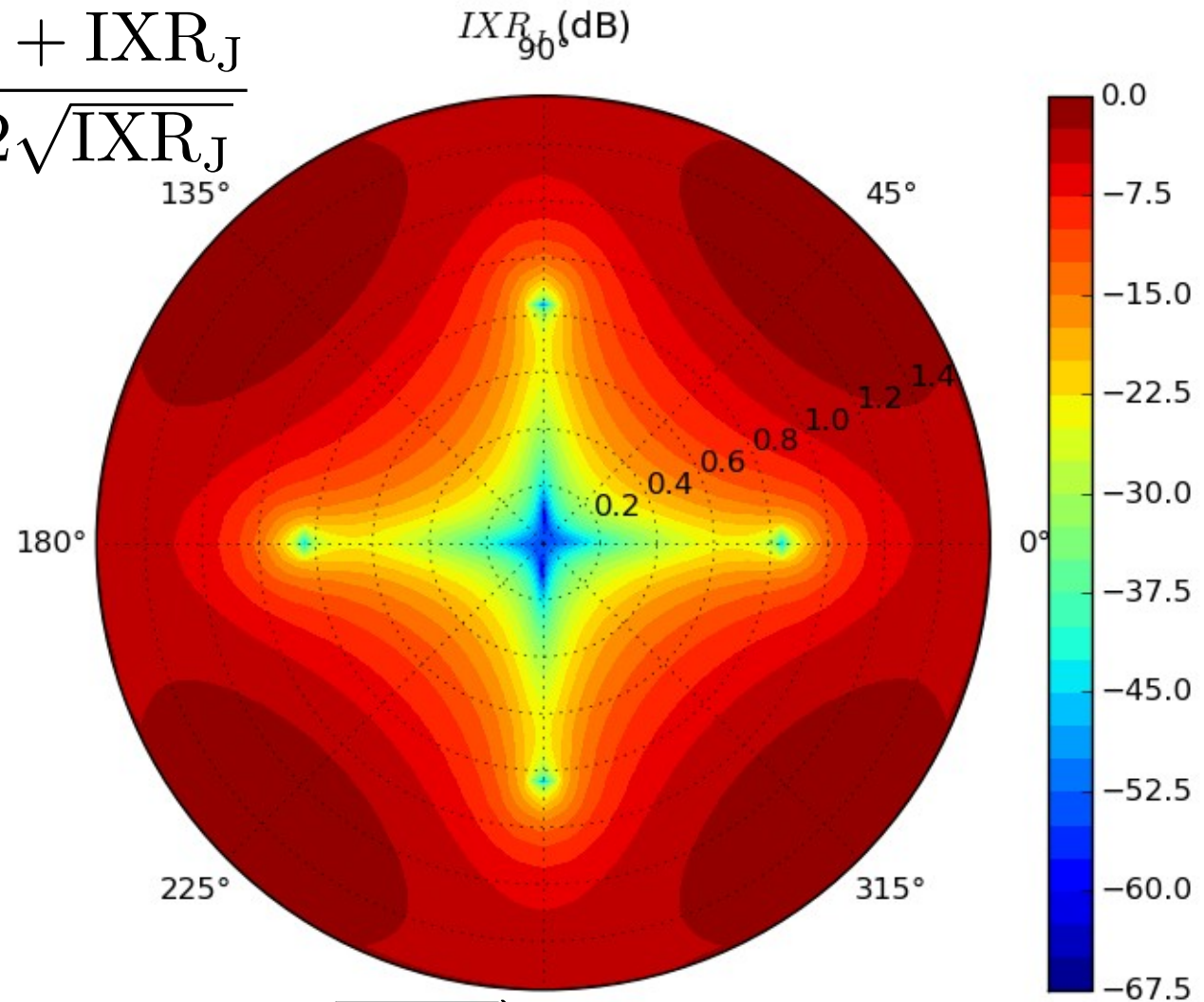
$$\text{IXR}_M = \frac{\kappa(M) + 1}{\kappa(M) - 1} = \frac{1 + \text{IXR}_J}{2\sqrt{\text{IXR}_J}}$$

$$\text{IXR}_J = \left(\frac{\kappa(J) + 1}{\kappa(J) - 1} \right)^2$$

$\kappa(M)$ Mueller and Jones matrix
 $\kappa(J)$ condition numbers

g_{\min} and g_{\max} : Minimum and maximum
 values when performing SVD

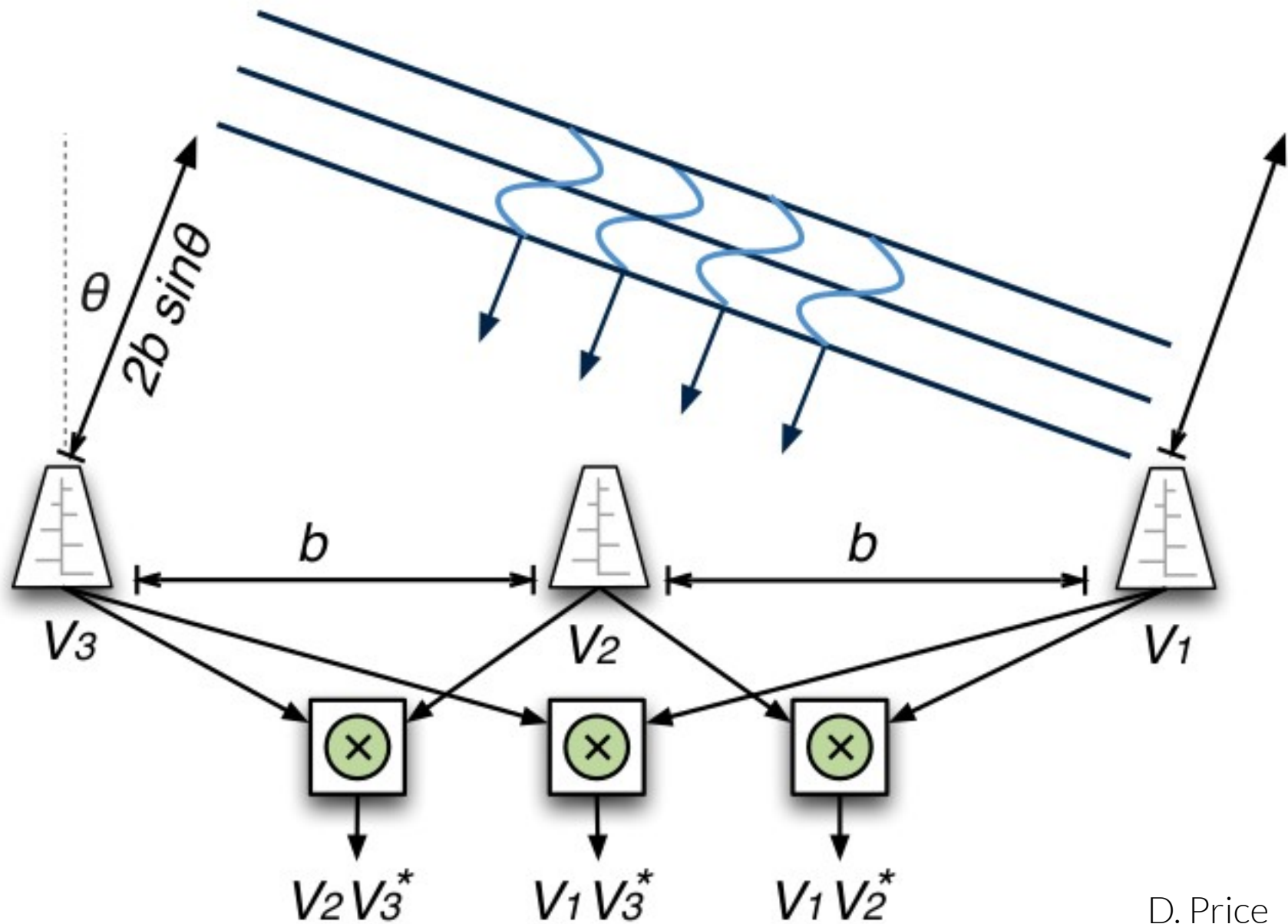
$$\mathbf{D} = \frac{g_{\max} + g_{\min}}{2} \begin{pmatrix} 1 & 1/\sqrt{\text{IXR}_J} \\ 1/\sqrt{\text{IXR}_J} & 1 \end{pmatrix}$$



PAPER Beam @ 110 MHz

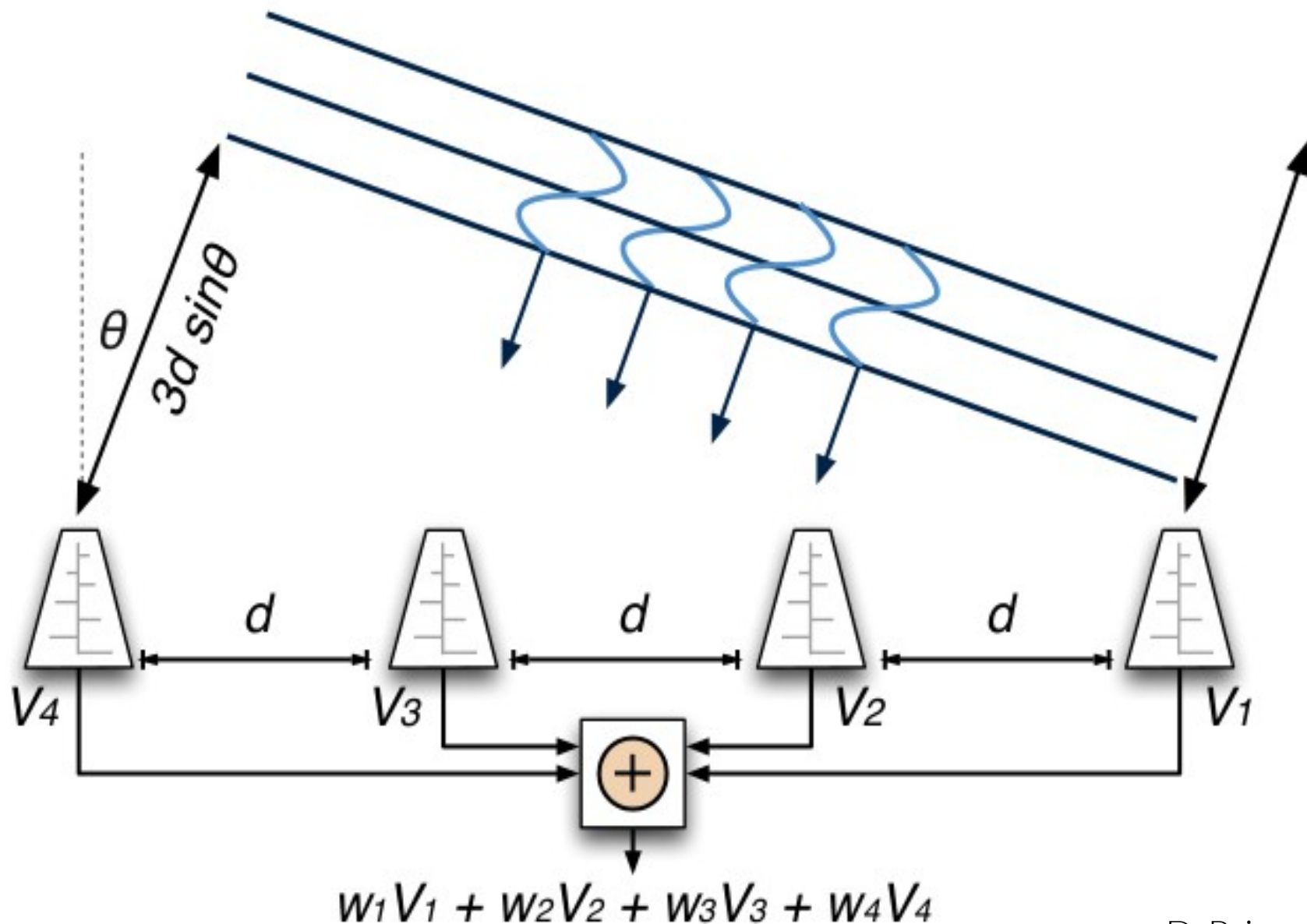
New Technologies

Simple Interferometric Model



D. Price

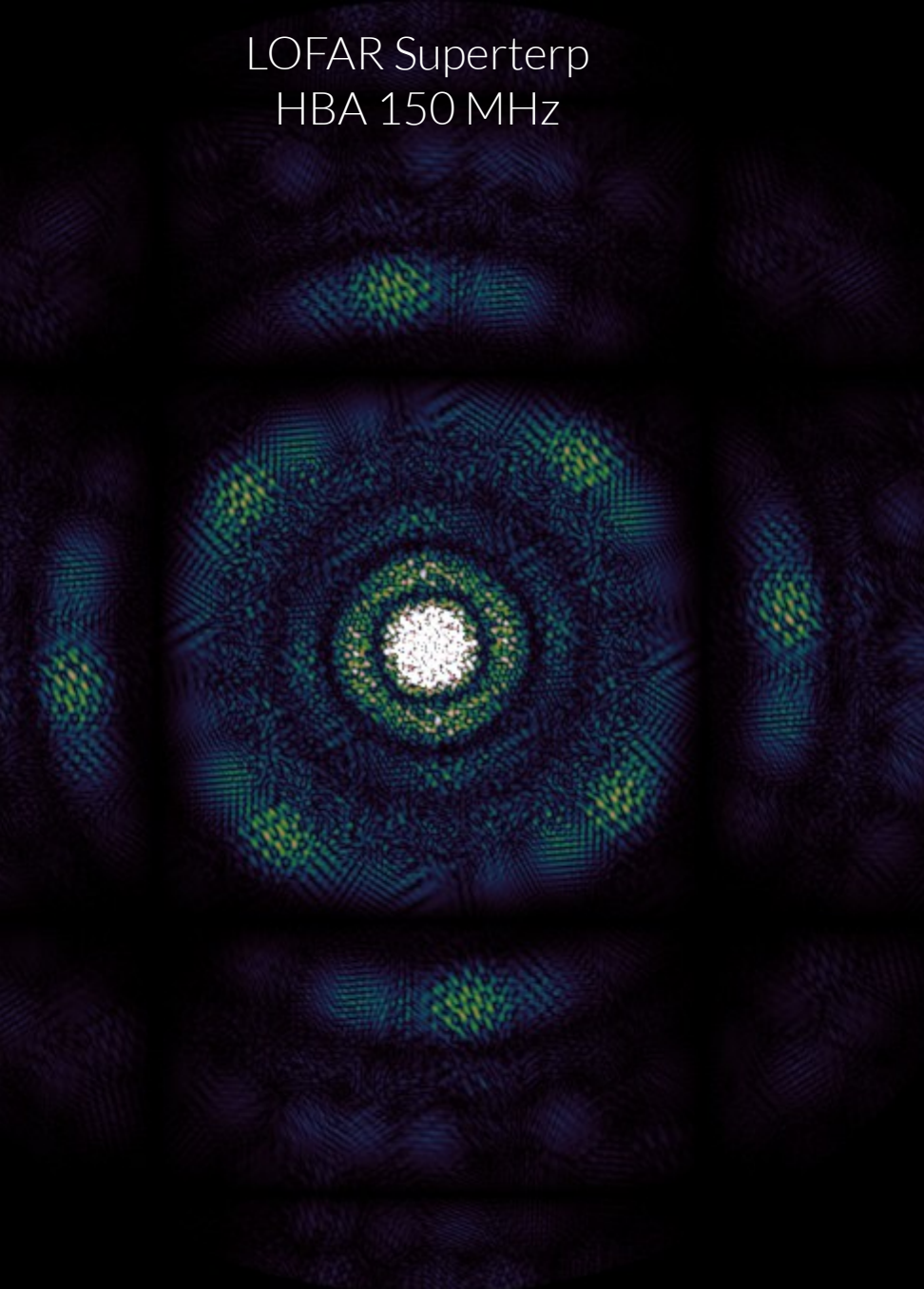
Simple Beamformer Model



D. Price

Beamformer Response

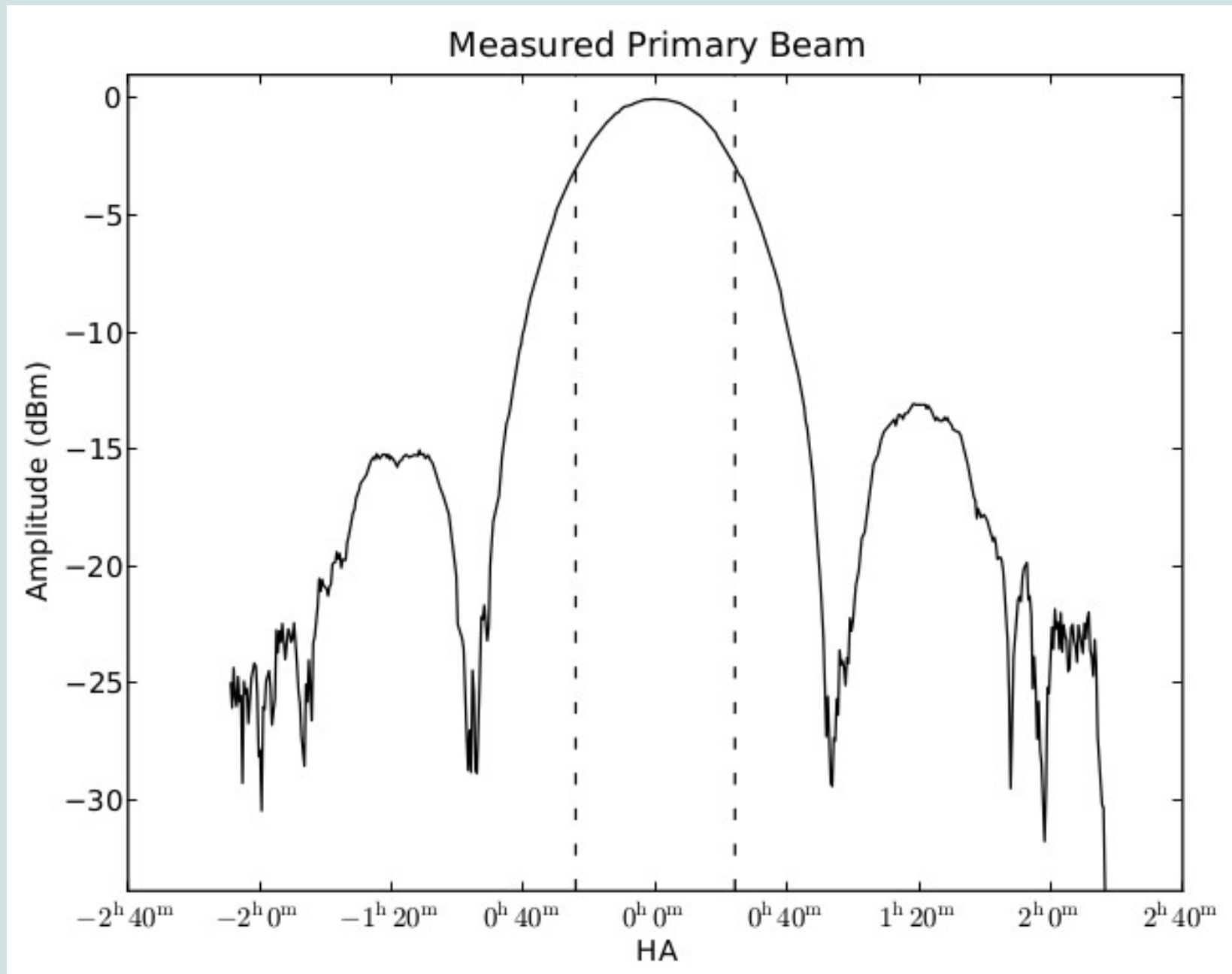
LOFAR Superterp
HBA 150 MHz



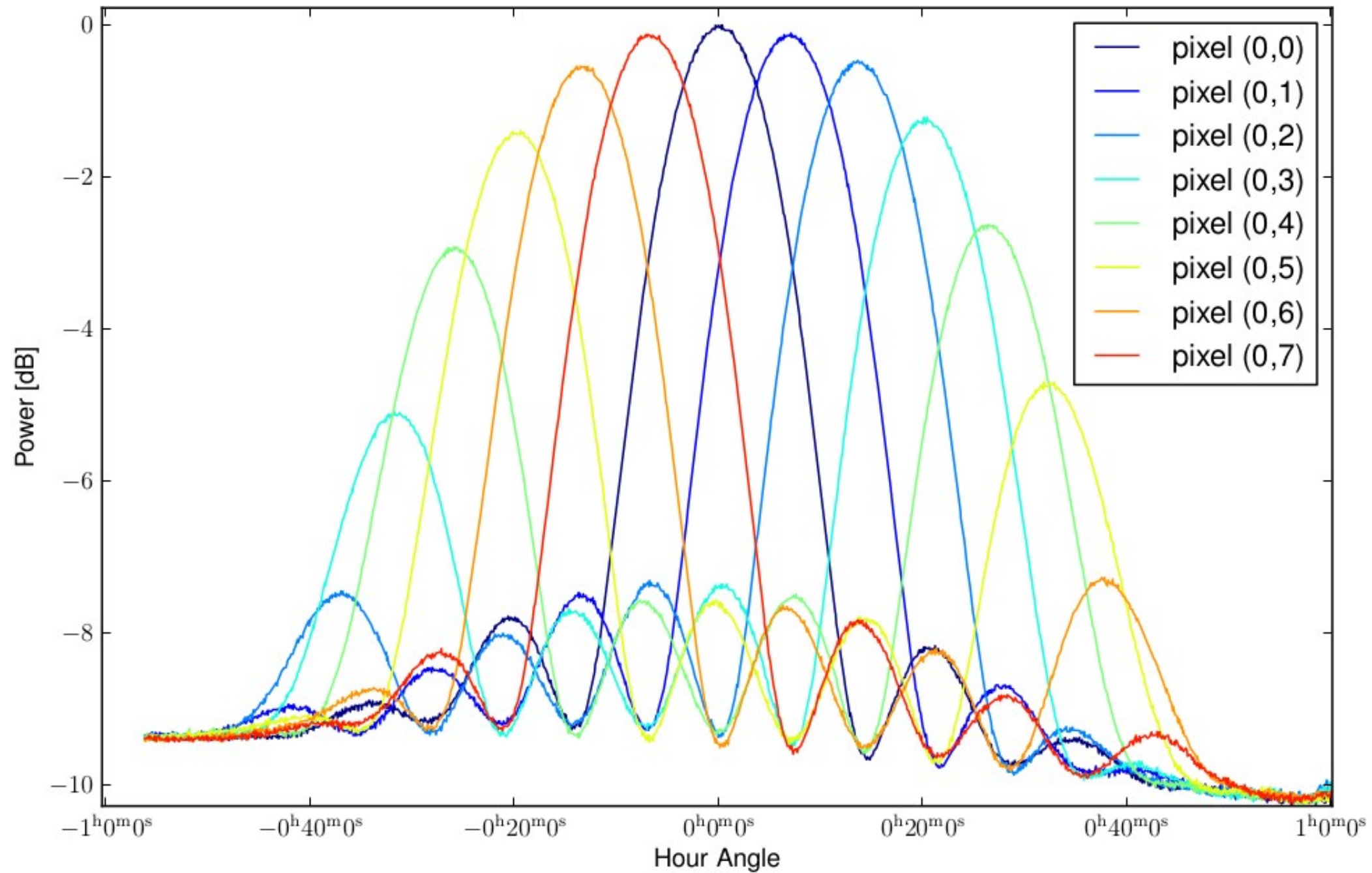
MWA Tile



Beamformer Response



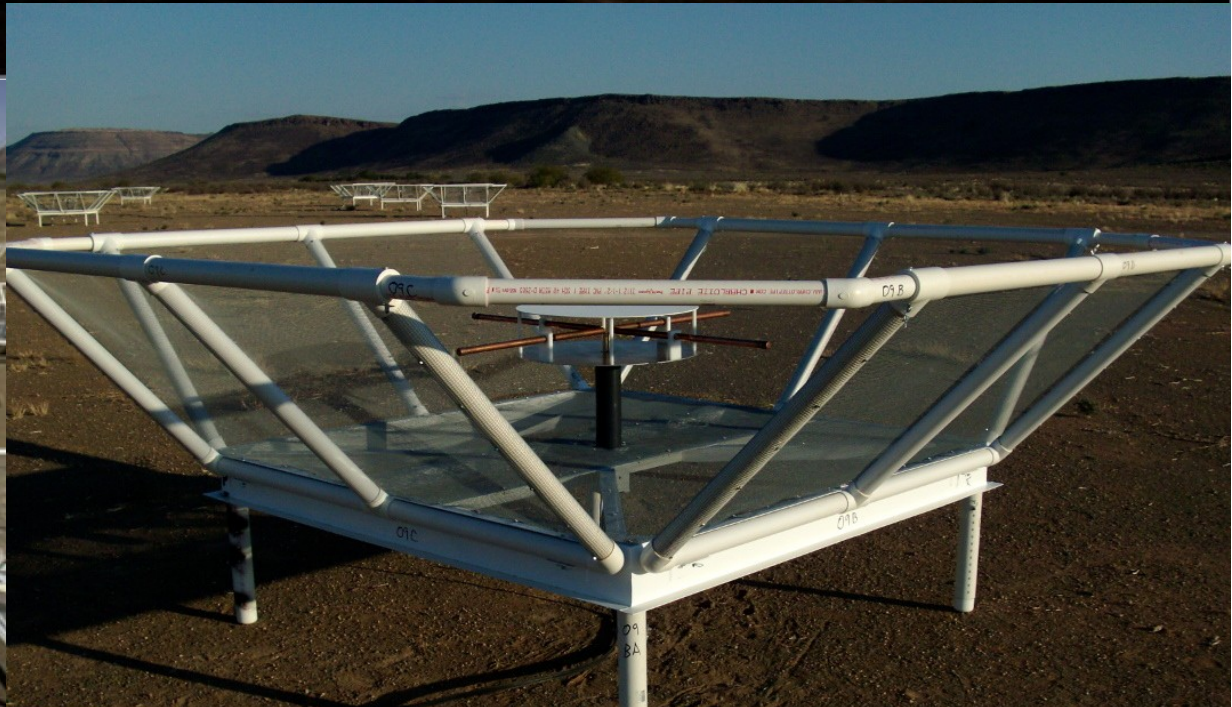
Beamformer Response



Phased Array Feeds (PAFs)



Transiting Arrays



Aperture Arrays



LOFAR Superterp

